# 優質教育基金
# 主題網絡計劃
# 2024/25

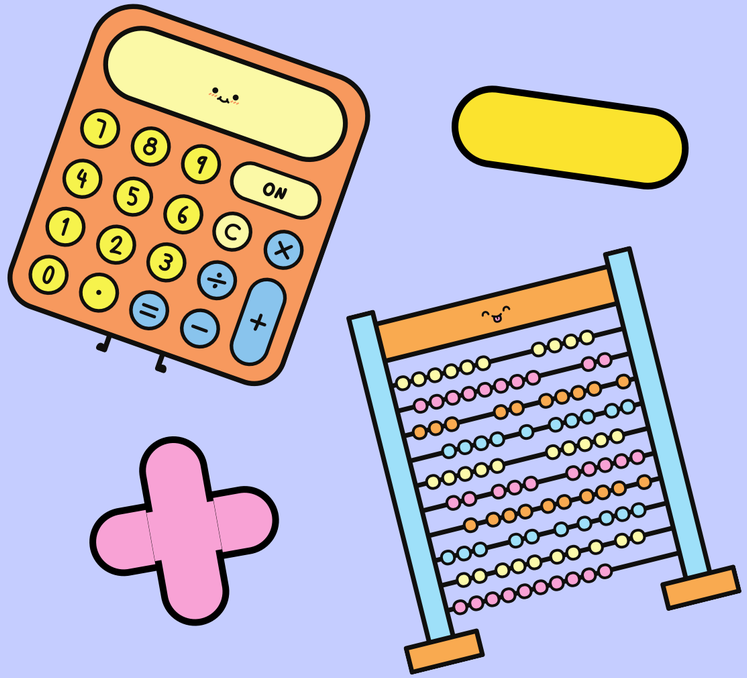
將數學建模注入中學數學科以推展STEAM教育

Promotion of STEAM Education by Infusing Mathematical Modelling into Secondary Mathematics

九龍真光中學 Kowloon True Light School
數學科 Mathematics Department

# 統籌及參與學校

- 九龍真光中學
- 伊利沙伯中學
- 可立中學（嗇色園主辦）
- 香港中文大學校友會聯會張煊昌中學



# 計劃目標

- 學生能運用數學知識及建模以解決問題
- 推動中學數學課程發展
- 培養教師在數學建模的認識及實踐

# 計劃特色

- 支援各參與學校設計並實行兩項數學建模活動
- 為參與教師舉辦數學建模講座
- 到校共同備課、觀課及評課
- 設立網上資源庫以分享各校的教材及成果
- 與教育局數學教育組及其他專業團體交流意見

# 本年度舉辦之活動

九龍真光中學

- 「如何推行數學建模」講座（2024-9-24）
- 「遊學團挑選方法」教材及課堂分享（2024-11-5）
- 設立網上資源庫及共享教案

# 各校舉辦之數學建模活動

伊利沙伯中學

- Dilution of household bleach: applying ratios and proportions to solve real-life problems (2024-12-11)
- Mathematics modelling project presentation (2025-3-21)
- Germany study tour (2025-5-21)

# 各校舉辦之數學建模活動

可立中學（嗇色園主辦）

- Introduction to mathematical modelling (2024-12-13)
- Exploring mathematical models from real life using probability (2025-5-2)

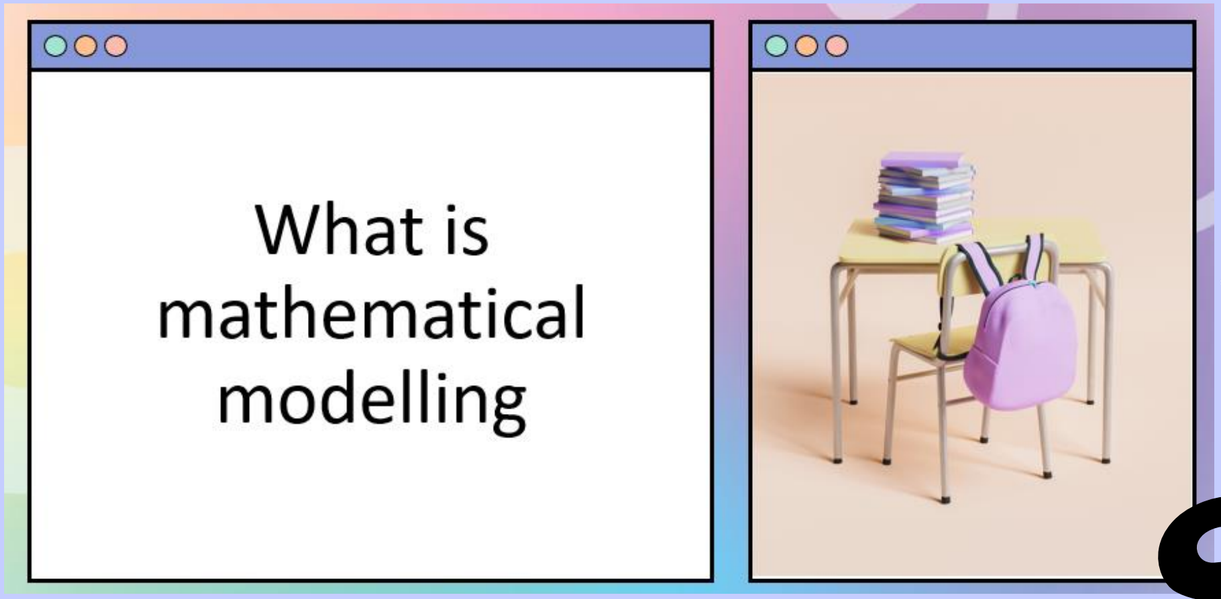
# 各校舉辦之數學建模活動

香港中文大學校友會聯會張煊昌中學

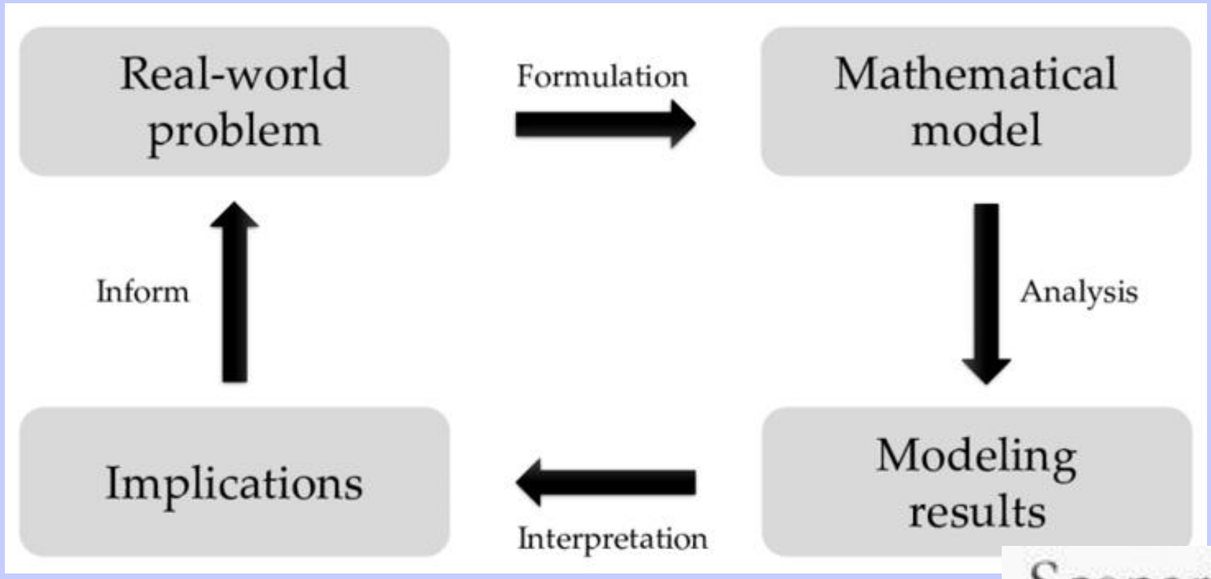
- 如何分配溫習時間（2024-12-17）
- 設計最划算的校外教學行程（2025-5-12）

# 「如何推行數學建模」講座

- 透過講座讓參與教師對數學建模初步認識
- 引用學者著作以鞏固數學建模教學基礎
- 分享事例令教師容易起步



# 「如何推行數學建模」講座

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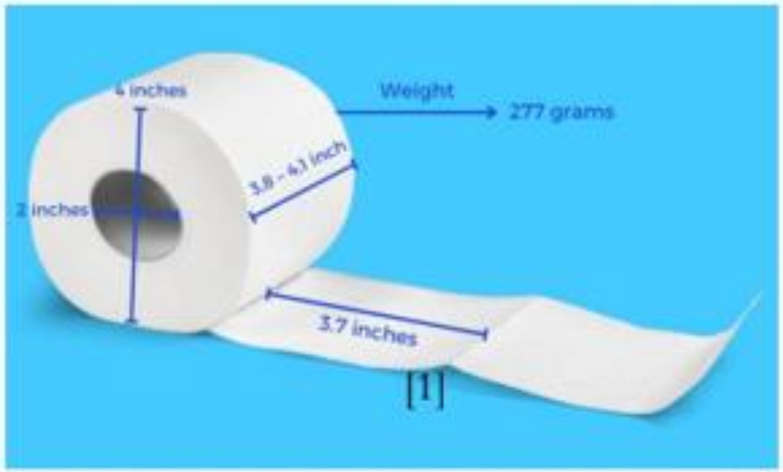


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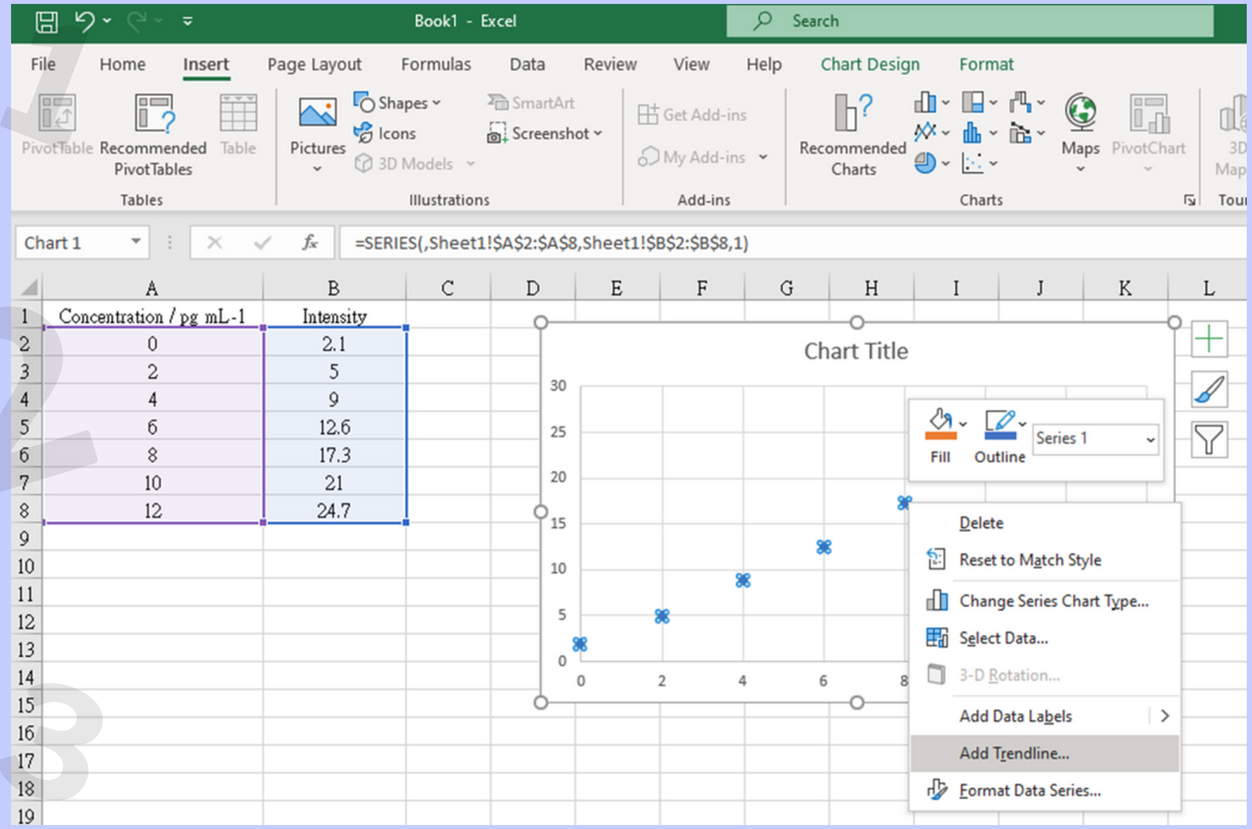
# 「如何推行數學建模」講座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Add Trendline...
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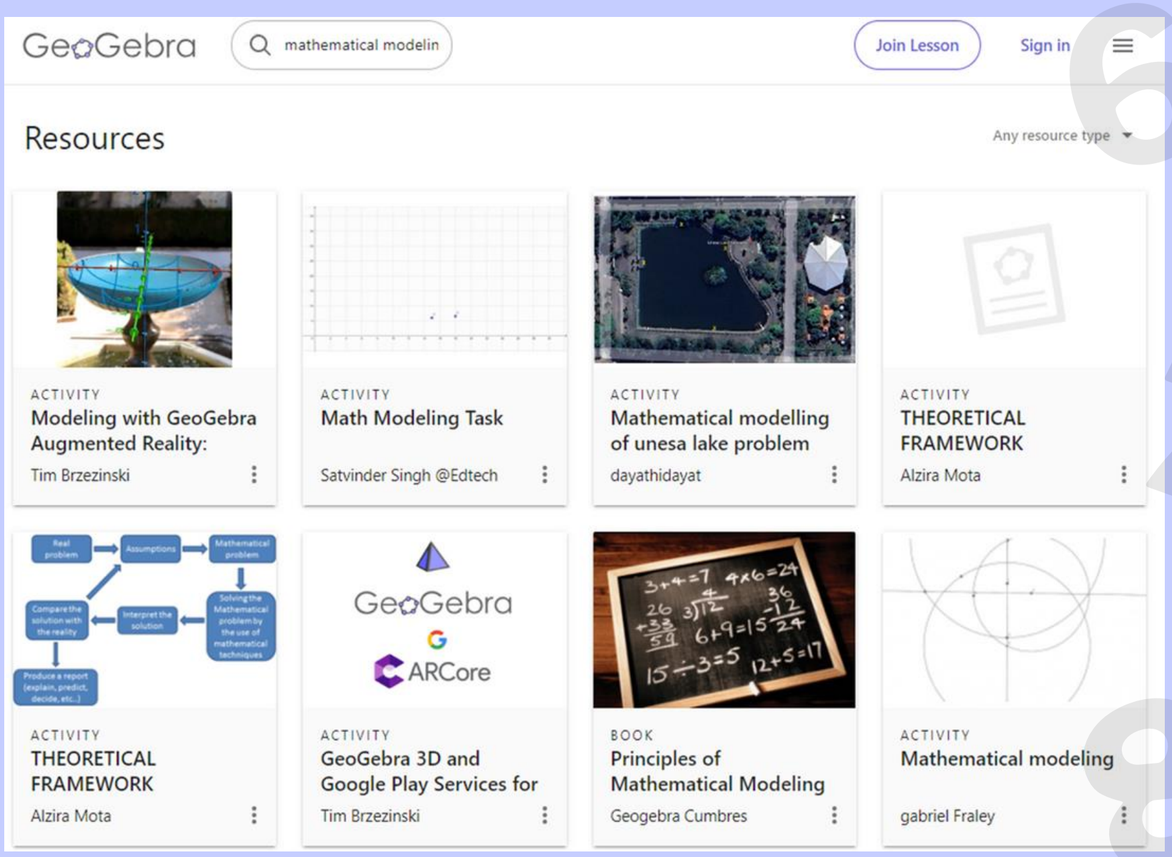
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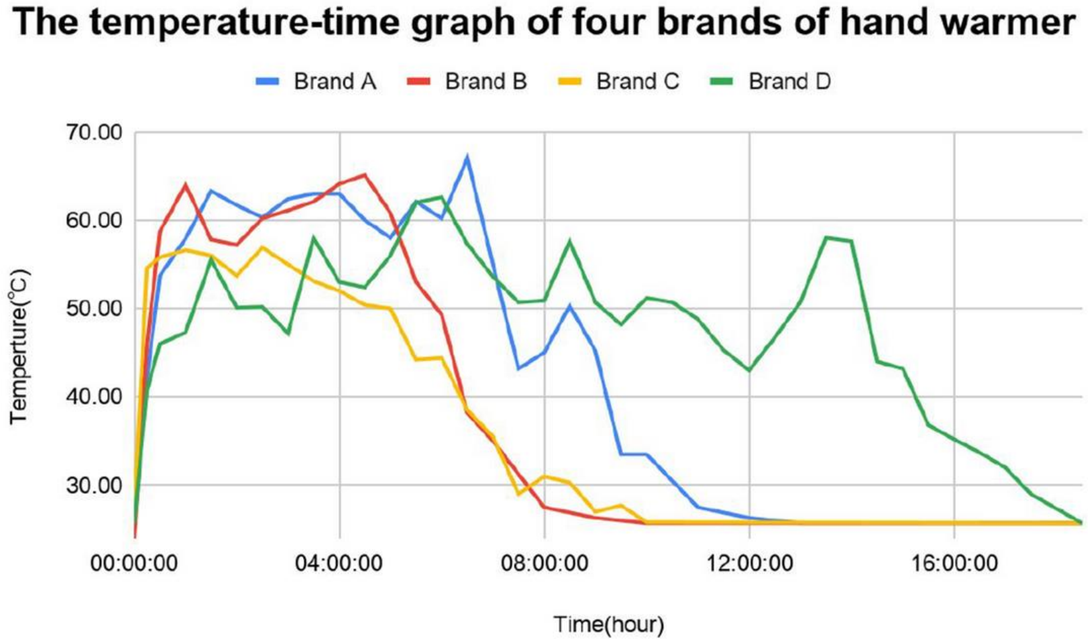
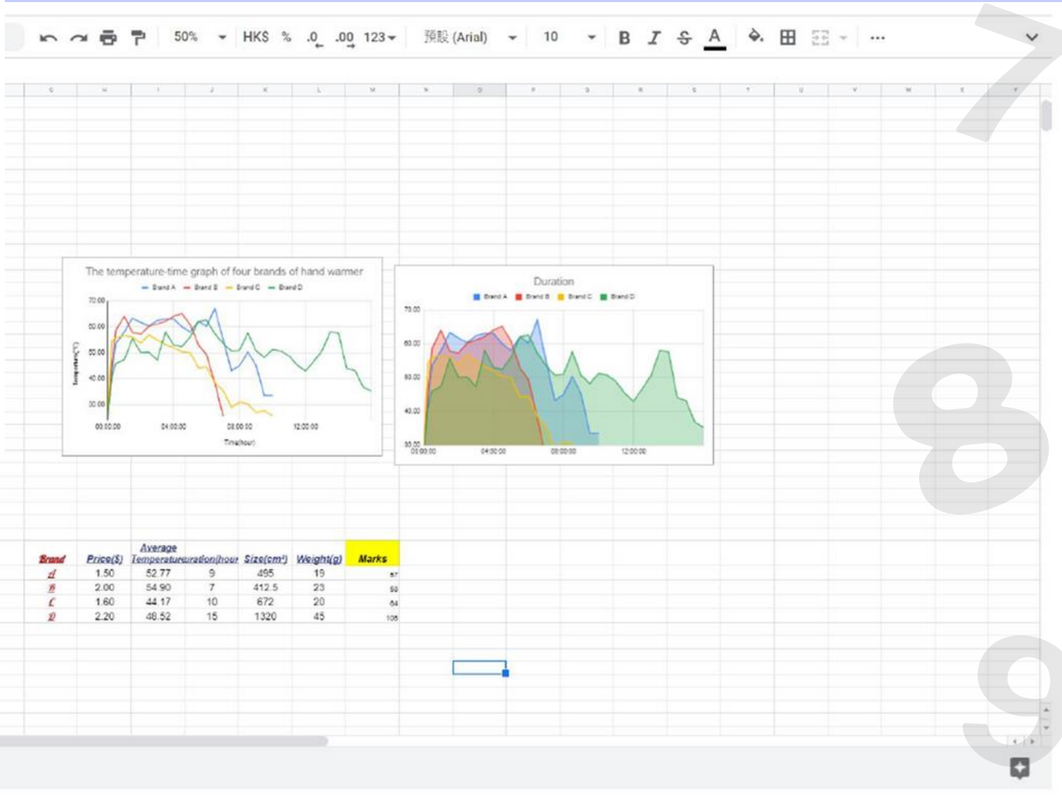
# 「如何推行數學建模」講座

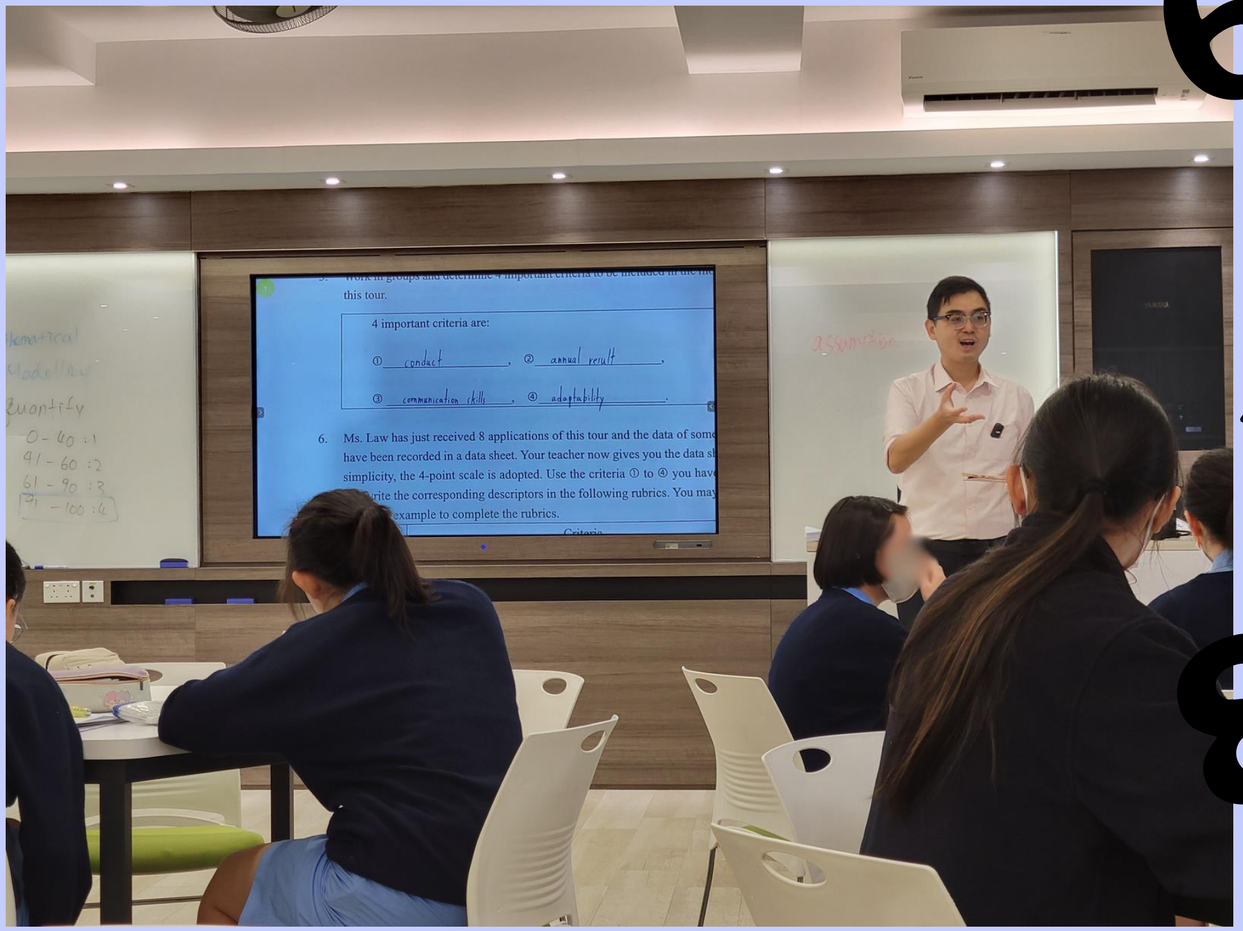
## The rate of change



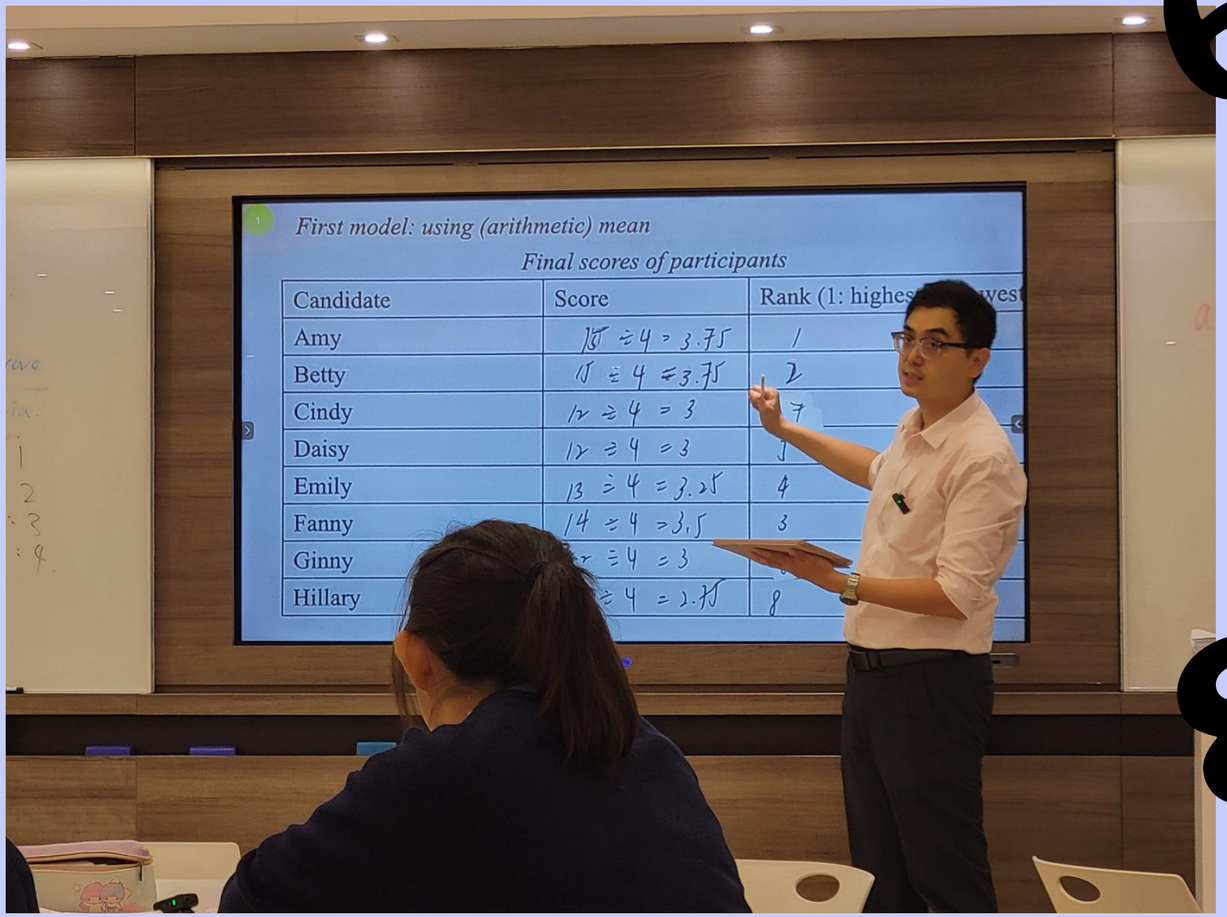
# 「遊學團挑選方法」教材及課堂分享

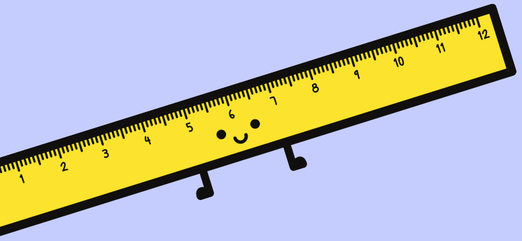
- 本校擔當領導者率先設計及實行數學建模活動
- 活動目的是為各參與學校「打強心針」
- 通過評課促進各校科組同事間專業交流

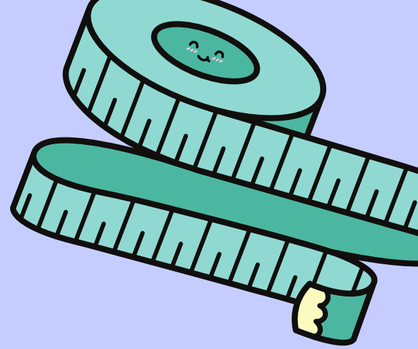


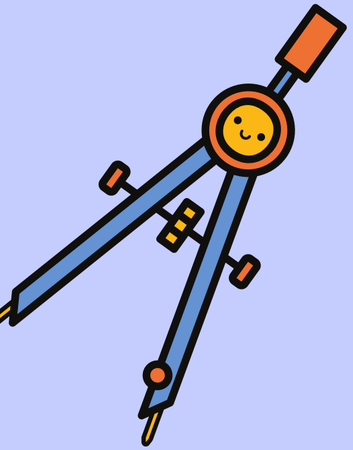
# 「遊學團挑選方法」教材及課堂感想

- 思考如何將數學建模融入現今課程內
- 活動的得益
- 學生能在課堂以外發揮數學建模的知識和技巧
- 跨學科協作（STEAM）



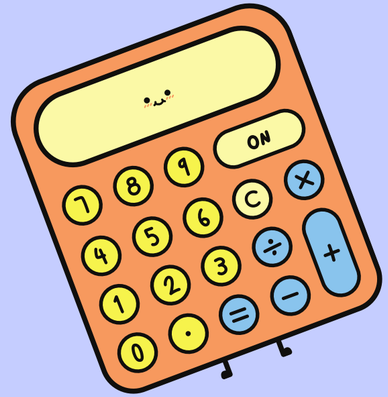


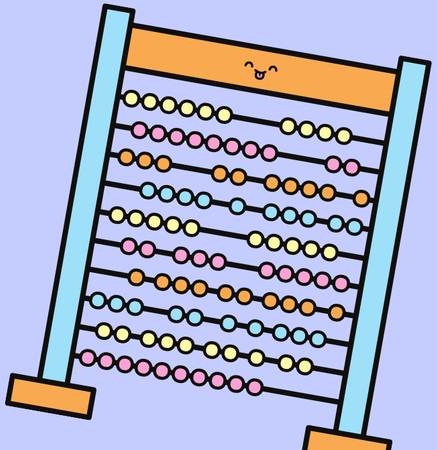




# 參與學校分享

伊利沙伯中學





# Queen Elizabeth School

## QEF Thematic Network on "Promotion of STEAM Education by Infusing Mathematical Modelling into Secondary Mathematics" (2024/25)

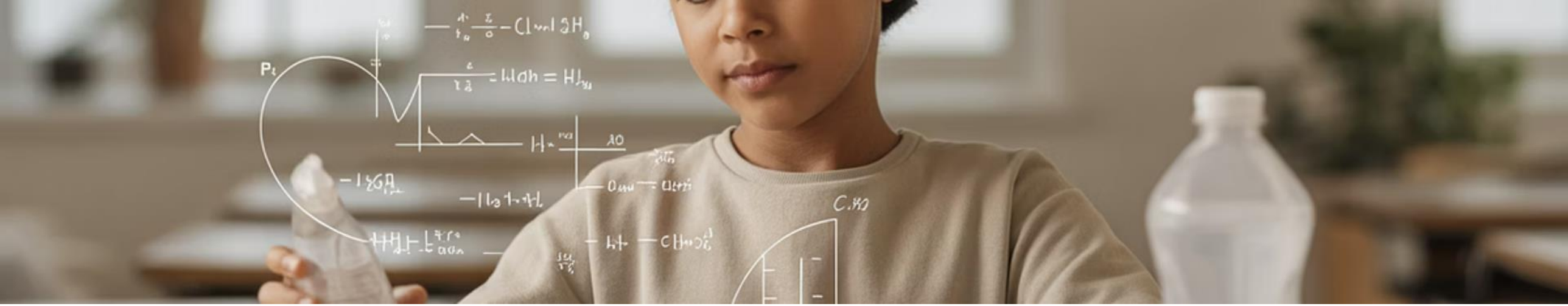
Mr. Marco Cheung, Ms. Wendy Cheung, Mr. Tommy Chiang

2025.06.17

# S2 Mathematical Modelling – Dilution of Household Bleach

Applying ratios and proportions to solve real-life problems

Date : 11 December 2024
Time : 09 : 55 – 11 : 15
Class: S.2B
Subject Teacher: Mr. Marco Cheung



# Project Overview

### Target Audience

Form 2 students,
Class of 34

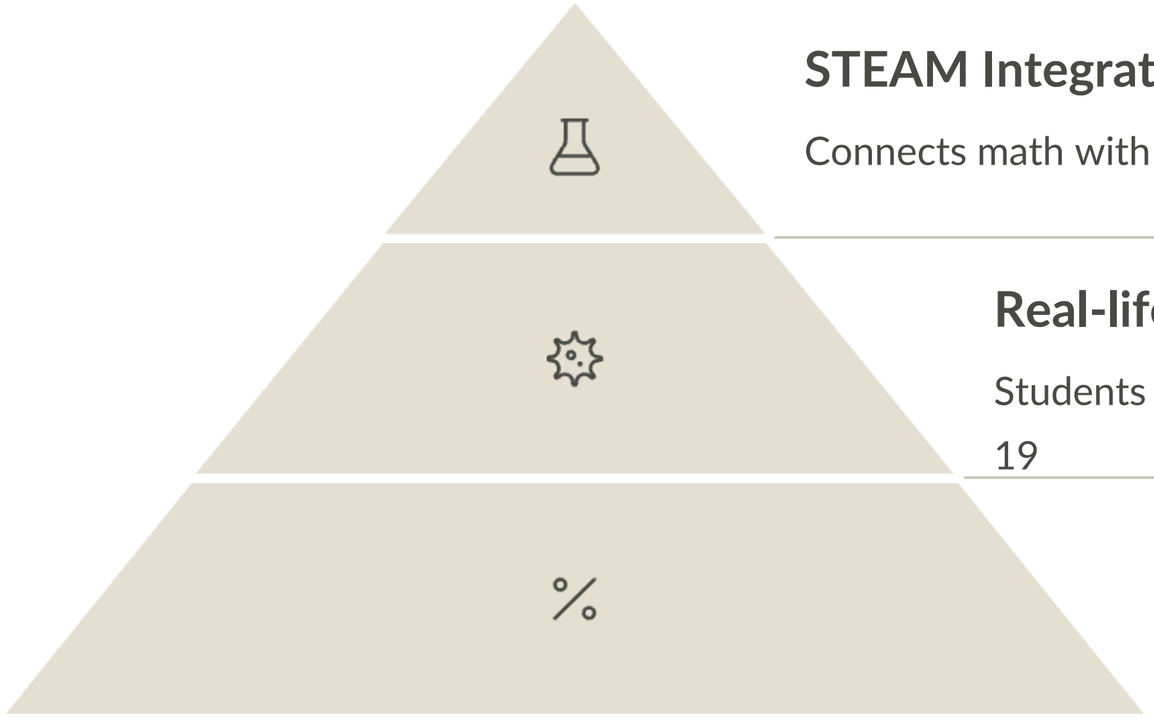
### Key Question

Which bleach performs best for specific purposes?

### Mathematical Focus

Applying ratios and proportions to real-life problems

# Why This Topic?



## STEAM Integration

Connects math with chemistry concepts

## Real-life Relevance

Students familiar with bleach from COVID-19

## Prior Knowledge

Builds on “Rates, Ratios and Proportions”

# Implementation Steps

## Problem Introduction

Connect to COVID experience

Present bleach comparison challenge

## Model Development

Derive a basic equation under ideal conditions

Guide criteria creation

## Excel Application

Input data into spreadsheets

Create ranking formulas

## Result Analysis

**Interpret findings**

**Discuss on the fairness of the result**


# Model Development Process

## Define Problem

Identify specific use case for bleach

## Create Criteria

Determine important performance factors

## Assign Weights

Apply proportional importance to each factor

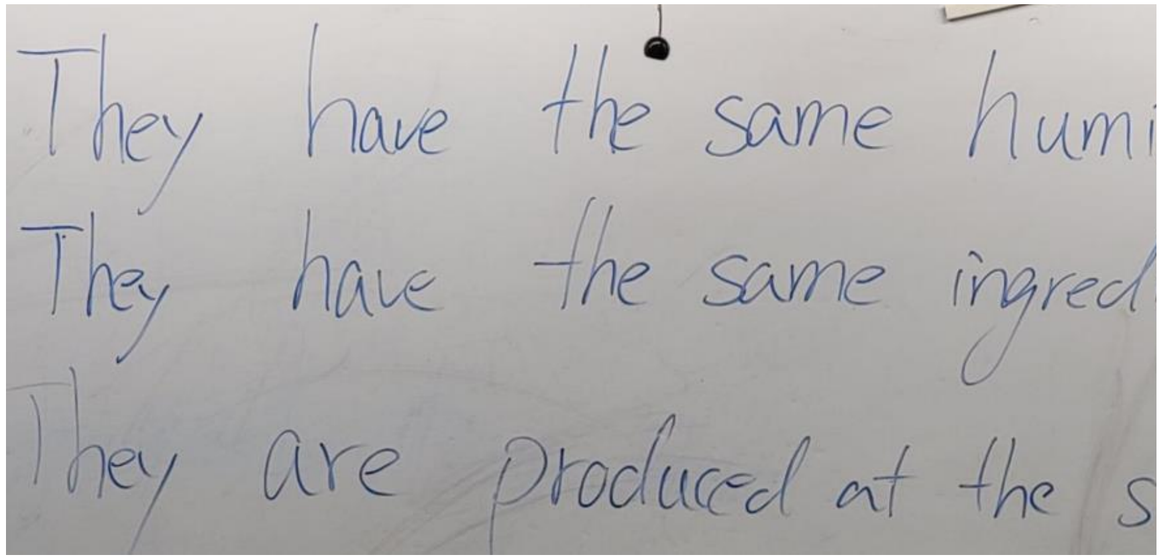
## Analyze Results

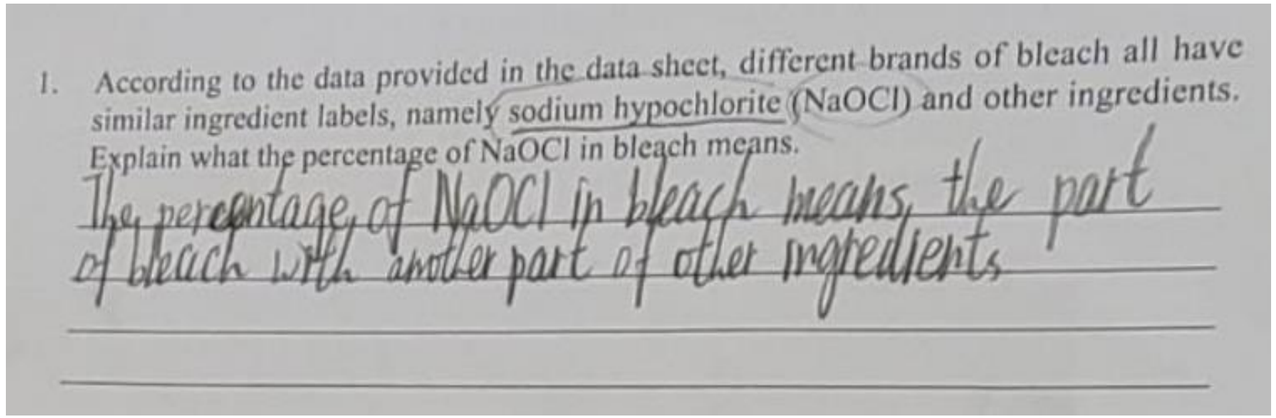
Rank bleach performance using Excel

# Student Observations

## What Worked Well

- Students able to create assumptions
- Successfully applied data to Excel models
- Students can analyze how different variables affect outcomes (Temperature, expire day)




## Challenges Faced

- Progress was slowed by unfamiliar chemical concepts
- Difficulty assigning fair weights to variables
- Some needed help refining models

# Improvement Suggestions

## Simplify Concepts

Choose topics without new concept

Saves valuable class time for modeling work

## Provide Context

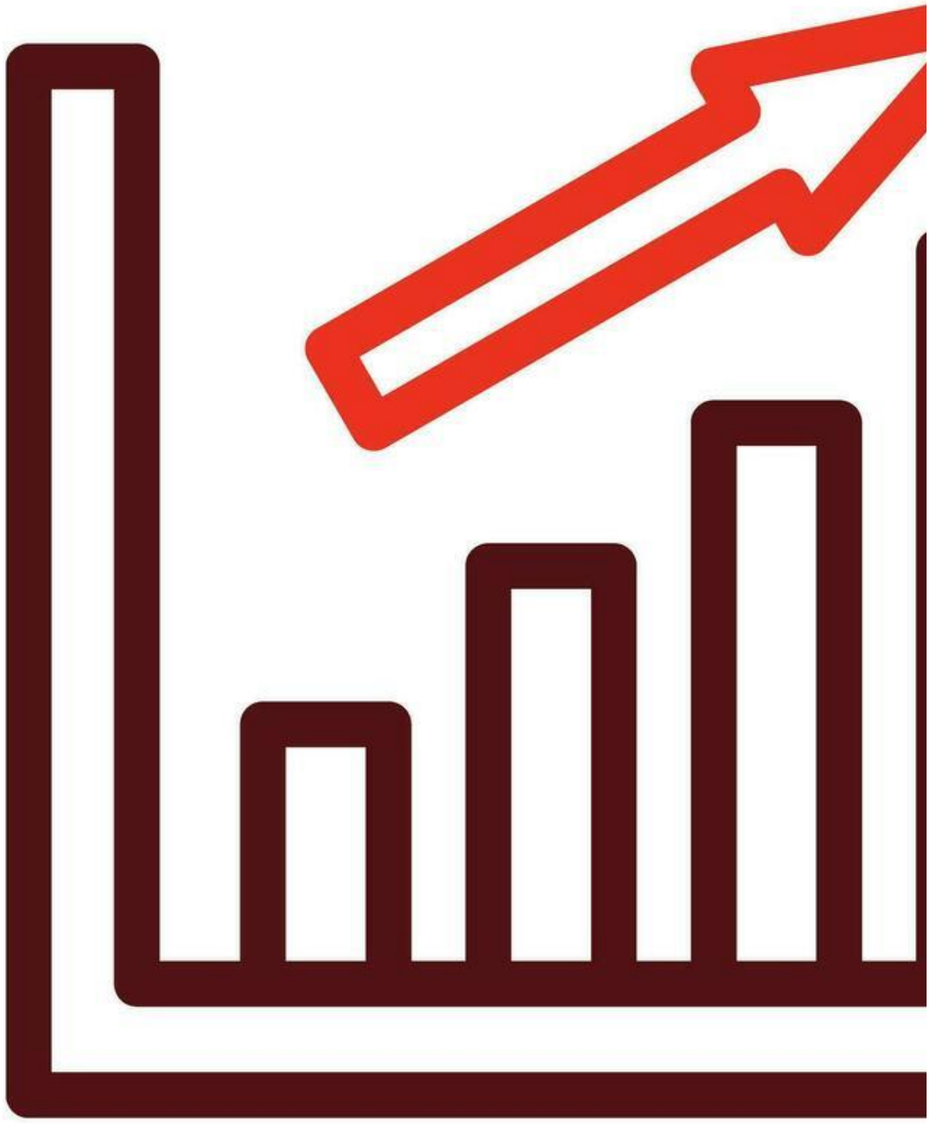
Give background on factors affecting performance

E.g. Temperature effects, Ingredient interactions

## Use Algebraic Notation

Replace words with variables: T = Temperature

Makes formulas clearer and more concise



# S.2 Mathematical Modelling

# Germany Study Tour

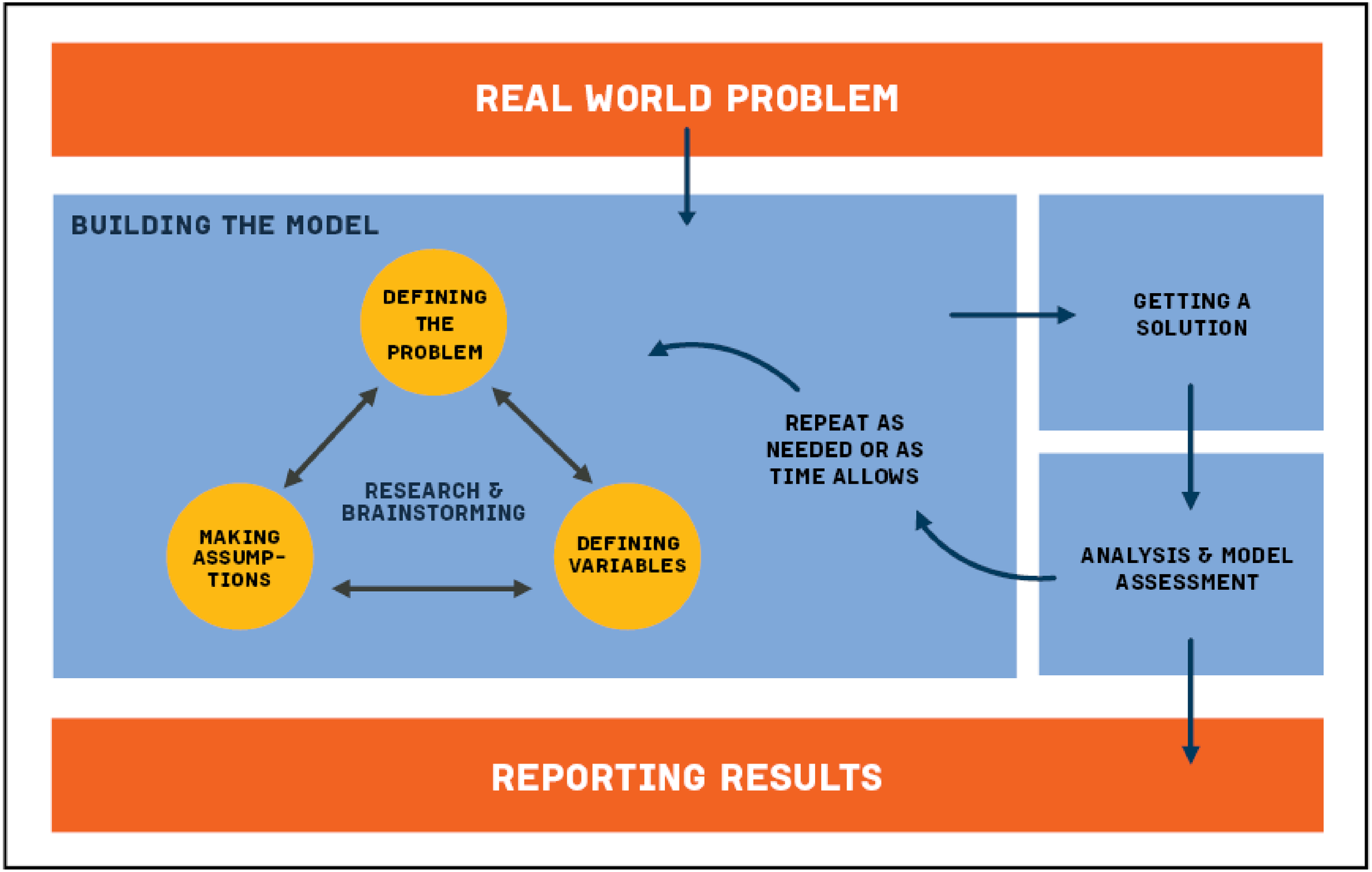
## Applying the weighted mean and proportion

Date : 21 May 2025

Time : 09 : 55 – 11 : 15

Class: S.2A

Teacher: Ms. Wendy Cheung


REAL WORLD PROBLEM
BUILDING THE MODEL
DEFINING THE PROBLEM
RESEARCH & BRAINSTORMING
MAKING ASSUMP-TIONS
DEFINING VARIABLES
REPEAT AS NEEDED OR AS TIME ALLOWS
GETTING A SOLUTION
ANALYSIS & MODEL ASSESSMENT
REPORTING RESULTS

# Define the Problem

Modeling problems are open-ended

Define with their 4 points scale for each group

5. Work in groups and determine 4 important criteria to be included in the models for this tour.

4 important criteria are:

① Annual result, ② Interview,

③ Attendance, ④ no. of languages

6. Ms. Cheung has received 8 applications of this tour and the data of some criteria have been recorded in a data sheet. Your teacher now gives you the data sheet. For simplicity, the 4-point scale is adopted. Use the criteria ① to ④ you have chosen and write the corresponding descriptors in the following rubrics. You may refer to the given example to complete the rubrics.

| | | Criteria | | | |
|---|---|---|---|---|---|
| Point scale | e.g. Conduct grade | ① Annual result | ② interview | ③ Attendance | ④ number of languages |
| 1 | D | 0-25 | 6 | less than 69 | 1 |
| 2 | C | 26-50 | 7 | 70-79 | 2 |
| 3 | B | 51-75 | 8 | 80-89 | 3 |
| 4 | A | 76-100 | 9 | 90-100 | 4 |

END OF TASK 1

# Make Assumptions

It is impossible to account for all the factors that impact a given situation,
so we must choose what is most important criteria to include in the study tour.


5. Work in groups and determine 4 important criteria to be included in the models for this tour.

4 important criteria are:
① Annual result, ② Interview,
③ Attendance, ④ no. of languages

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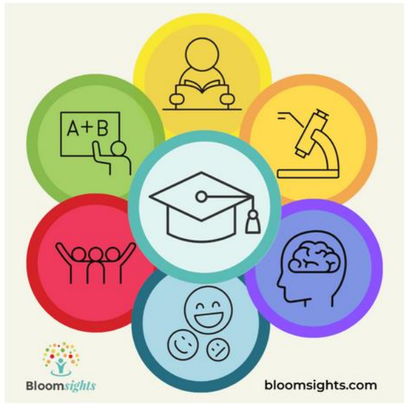
END OF TASK 1



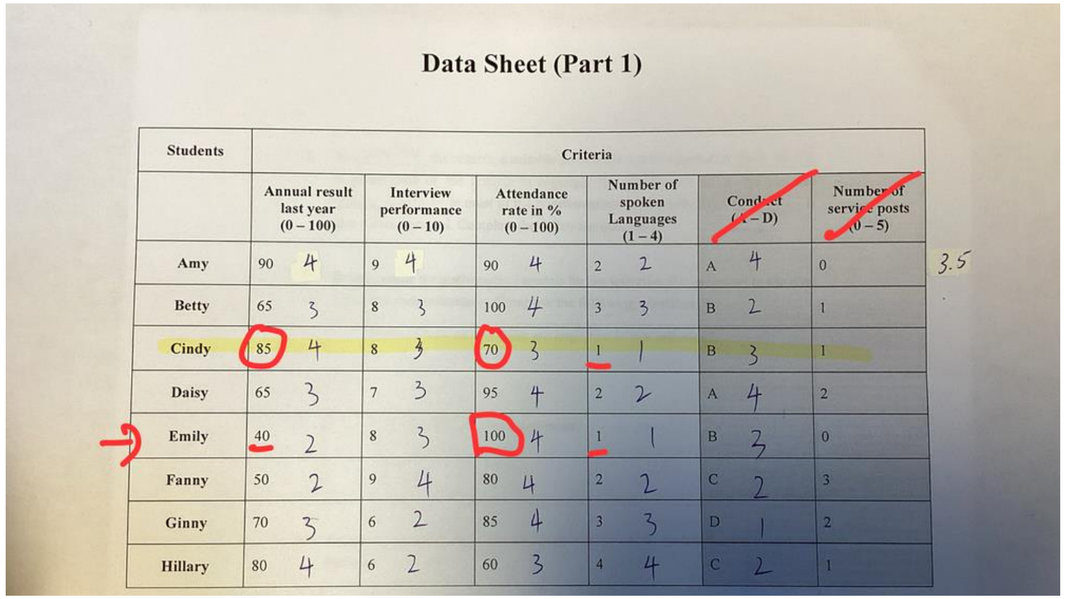

# Define Variables

Making assumptions reveals variables that will be needed.

Each group defines each weights for each criteria.

Weights become variables among different groups.

| Students | Criteria | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Annual result last year (0 – 100) | Interview performance (0 – 10) | Attendance rate in % (0 – 100) | Number of spoken Languages (1 – 4) | Conduct (A – D) | Number of service posts (0 – 5) | Feedback from teachers (Poor, Fair, Good, Excellent) | Joined tour before? Which year? Which tour? |
| Amy | 90 | 9 | 90 | 2 | A | 0 | Good | 2023 Beijing tour |
| Betty | 65 | 8 | 100 | 3 | B | 1 | Excellent | 2024 Beijing tour |
| Cindy | 85 | 8 | 70 | 1 | B | 1 | Poor | Never |
| Daisy | 65 | 7 | 95 | 2 | A | 2 | Fair | 2024 UK tour |
| Emily | 40 | 8 | 100 | 1 | B | 0 | Good | Never |
| Fanny | 50 | 9 | 80 | 2 | C | 3 | Excellent | Never |
| Ginny | 70 | 6 | 85 | 3 | D | 2 | Good | 2023 Japan tour |
| Hillary | 80 | 6 | 60 | 4 | C | 1 | Fair | 2024 Beijing tour |



Data Sheet (Part 1)

| Students | Criteria | | | | | |
|---|---|---|---|---|---|---|
| | Annual result last year (0 – 100) | Interview performance (0 – 10) | Attendance rate in % (0 – 100) | Number of spoken Languages (1 – 4) | Conduct (A – D) | Number of service posts (0 – 5) |
| Amy | 90 4 | 9 4 | 90 4 | 2 2 | A 4 | 0 |
| Betty | 65 3 | 8 3 | 100 4 | 3 3 | B 2 | 1 |
| Cindy | 85 4 | 8 3 | 70 3 | 1 1 | B 3 | 1 |
| Daisy | 65 3 | 7 3 | 95 4 | 2 2 | A 4 | 2 |
| Emily | 40 2 | 8 3 | 100 4 | 1 1 | B 3 | 0 |
| Fanny | 50 2 | 9 4 | 80 4 | 2 2 | C 2 | 3 |
| Ginny | 70 3 | 6 2 | 85 4 | 3 3 | D 1 | 2 |
| Hillary | 80 4 | 6 2 | 60 3 | 4 4 | C 2 | 1 |

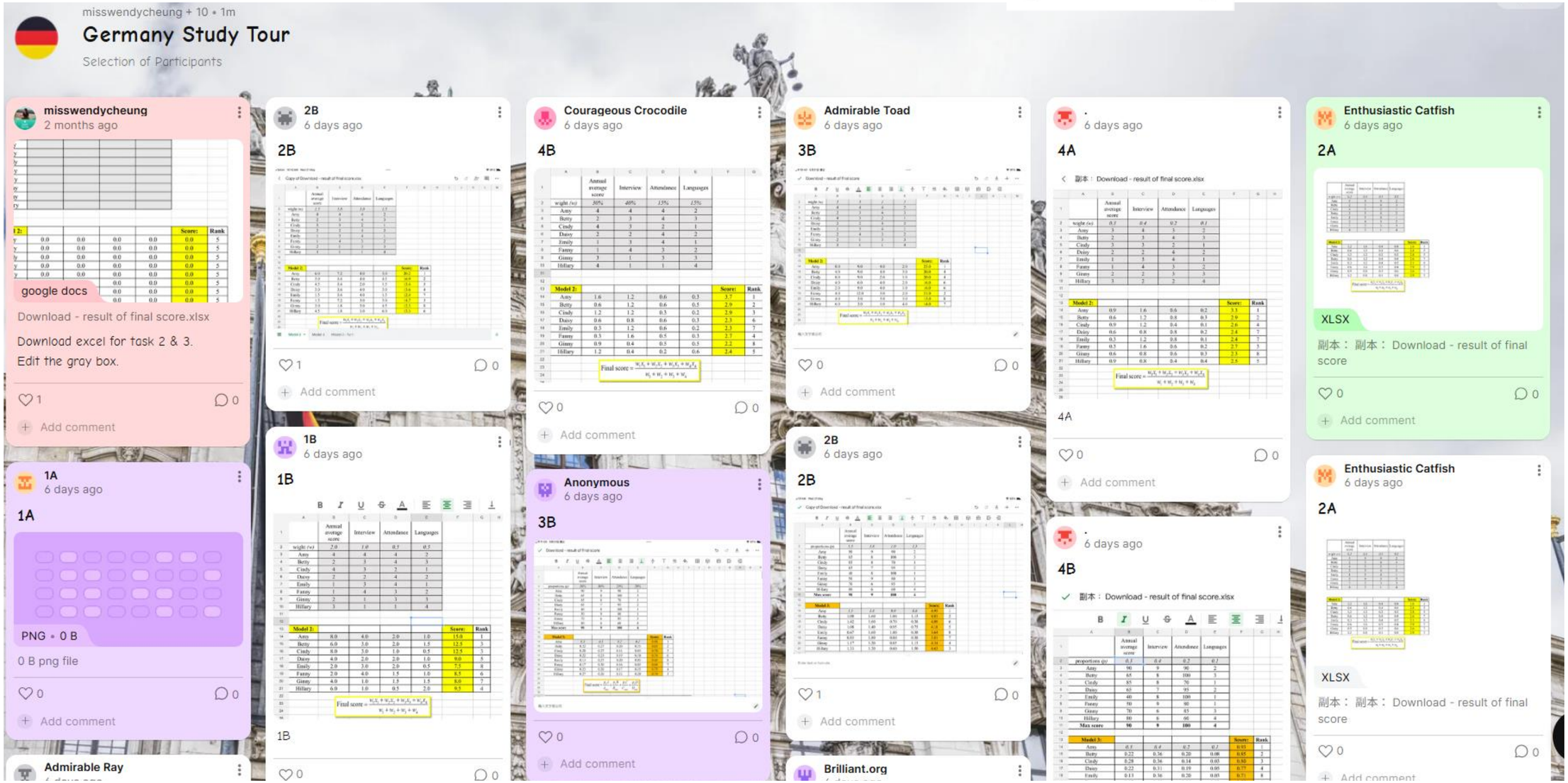
3.5

# Get a Solution

Use familiar mathematical and computational tools to solve..

Using google sheet/ excel as tools to calculate rapidly.

Padlet



# Model 1 – Average
# Model 2 – Weighted mean

## Group A

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | | Annual average score | Interview | Attendance | Languages |
| 2 | weight *(w)* | *30%* | *40%* | *15%* | *15%* |
| 3 | Amy | 4 | 4 | 4 | 2 |
| 4 | Betty | 2 | 3 | 4 | 3 |
| 5 | Cindy | 4 | 3 | 2 | 1 |
| 6 | Daisy | 2 | 2 | 4 | 2 |
| 7 | Emily | 1 | 3 | 4 | 1 |
| 8 | Fanny | 1 | 4 | 3 | 2 |
| 9 | Ginny | 3 | 1 | 3 | 3 |
| 10 | Hillary | 4 | 1 | 1 | |
| 11 | | | | | |
| 12 | | | | | |
| 13 | **Model 2:** | | | | |
| 14 | Amy | 1.6 | 1.2 | 0.6 | |
| 15 | Betty | 0.6 | 1.2 | 0.6 | |
| 16 | Cindy | 1.2 | 1.2 | 0.3 | 0.2 |
| 17 | Daisy | 0.6 | 0.8 | 0.6 | 0.3 |
| 18 | Emily | 0.3 | 1.2 | 0.6 | 0.2 |
| 19 | Fanny | 0.3 | 1.6 | 0.5 | 0.3 |
| 20 | Ginny | 0.9 | 0.4 | 0.5 | 0.5 |
| 21 | Hillary | 1.2 | 0.4 | 0.2 | 0.6 |
| 22 | | | | | |
| 23 | | | | | |
| 24 | | | | | |

$$\text{Final score} = \frac{w_1x_1 + w_2x_2 + w_3x_3 + w_4x_4}{w_1 + w_2 + w_3 + w_4}$$

Different weighting

Different 4 point scale standard

## Group B

| | Annual average score | Interview | Attendance | Languages |
|---|---|---|---|---|
| weight *(w)* | *0.3* | *0.4* | *0.1* | *0.2* |
| Amy | 4 | 4 | 4 | 2 |
| Betty | 2 | 3 | 4 | 3 |
| Cindy | 4 | 3 | 2 | 1 |
| Daisy | 2 | 3 | 4 | 2 |
| Emily | 1 | 3 | 4 | 1 |
| Fanny | 2 | 4 | 3 | 2 |
| Ginny | 3 | 2 | 3 | 3 |
| Hillary | 4 | 2 | 1 | 4 |

| | | | | | Score: | Rank |
|---|---|---|---|---|---|---|
| | 1.2 | 1.6 | 0.4 | 0.4 | 3.6 | 1 |
| | 0.6 | 1.2 | 0.4 | 0.6 | 2.8 | 5 |
| Cindy | 1.2 | 1.2 | 0.2 | 0.2 | 2.8 | 4 |
| Daisy | 0.6 | 1.2 | 0.4 | 0.4 | 2.6 | 7 |
| Emily | 0.3 | 1.2 | 0.4 | 0.2 | 2.1 | 8 |
| Fanny | 0.6 | 1.6 | 0.3 | 0.4 | 2.9 | 3 |
| Ginny | 0.9 | 0.8 | 0.3 | 0.6 | 2.6 | 7 |
| Hillary | 1.2 | 0.8 | 0.1 | 0.8 | 2.9 | 2 |

$$\text{Final score} = \frac{w_1x_1 + w_2x_2 + w_3x_3 + w_4x_4}{w_1 + w_2 + w_3 + w_4}$$

Different results of ranking

# Analyze the Solution

When considering the results and insights gained from the model, ask how to improve from the model.
Why is the different results from different group?

1. You have learnt the (arithmetic) mean in the textbook. Try to use that formula as your first model to find out the final scores of the 8 applications following the rubrics you have set in the previous task.

*First model: using (arithmetic) mean*

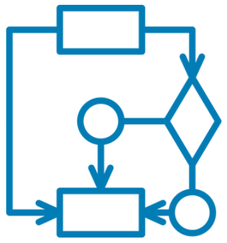
*Final scores of participants*

| Candidate | Score | Rank (1: highest, 8: lowest) |
|---|---|---|
| Amy | 3.5 | 1 |
| Betty | 3.25 | 2 |
| Cindy | 2.5 | 4 |
| Daisy | 2.75 | 3 |
| Emily | 2.5 | 4 |
| Fanny | 2.25 | 8 |
| Ginny | 2.5 | 4 |
| Hillary | 2.5 | 4 |

We need to justify whether the model is possible for the selection.
Try to answer the following questions:

- What is the advantage of this formula?

Easy, fast



# Iterate

*Refine the model by repeating the process, adjusting as needed to improve the solution.*

Instead of repeating the model, we compare the different results with the same model among different groups to save time.

## Group C

✓ 副本： Download - result of final score.xlsx

**Model 3 – Weighted mean with Amax, Bmax, Cmax, Dmax**

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | | Annual average score | Interview | Attendance | Languages | | |
| 2 | proportions *(p)* | *0.3* | *0.4* | *0.2* | *0.1* | | |
| 3 | Amy | 90 | 9 | 90 | 2 | | |
| 4 | Betty | 65 | 8 | 100 | 3 | | |
| 5 | Cindy | 85 | 8 | 70 | 1 | | |
| 6 | Daisy | 65 | 7 | 95 | 2 | | |
| 7 | Emily | 40 | 8 | 100 | 1 | | |
| 8 | Fanny | 50 | 9 | 80 | 1 | | |
| 9 | Ginny | 70 | 6 | 85 | 3 | | |
| 10 | Hillary | 80 | 6 | 60 | 4 | | |
| 11 | **Max score** | **90** | **9** | **100** | **4** | | |
| 12 | | | | | | | |
| 13 | **Model 3:** | | | | | **Score:** | **Rank** |
| 14 | Amy | *0.3* | *0.4* | *0.2* | *0.1* | 0.93 | 1 |
| 15 | Betty | 0.22 | 0.36 | 0.20 | 0.08 | 0.85 | 2 |
| 16 | Cindy | 0.28 | 0.36 | 0.14 | 0.03 | 0.80 | 3 |
| 17 | Daisy | 0.22 | 0.31 | 0.19 | 0.05 | 0.77 | 4 |
| 18 | Emily | 0.13 | 0.36 | 0.20 | 0.03 | 0.71 | 8 |
| 19 | Fanny | 0.17 | 0.40 | 0.16 | 0.03 | 0.75 | 6 |
| 20 | Ginny | 0.23 | 0.27 | 0.17 | 0.08 | 0.75 | 7 |
| 21 | Hillary | 0.27 | 0.27 | 0.12 | 0.10 | 0.75 | 5 |

$$\text{Final score} = \frac{p_1 A}{A_{\max}} + \frac{p_2 B}{B_{\max}} + \frac{p_3 C}{C_{\max}} + \frac{p_4 D}{D_{\max}}$$

## Group D

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | | Annual average score | Interview | Attendance | Languages | | |
| 2 | proportions *(p)* | *30%* | *30%* | *20%* | *20%* | | |
| 3 | Amy | 90 | 9 | 90 | 2 | | |
| 4 | Betty | 65 | 8 | 100 | 3 | | |
| 5 | Cindy | 85 | 8 | 70 | 1 | | |
| 6 | Daisy | 65 | 7 | 95 | 2 | | |
| 7 | Emily | 40 | 8 | 100 | 1 | | |
| 8 | Fanny | 50 | 9 | 80 | 1 | | |
| 9 | Ginny | 70 | 6 | 85 | 3 | | |
| 10 | Hillary | 80 | 6 | 60 | 4 | | |
| 11 | **Max score** | **90** | **9** | **100** | **4** | | |
| 12 | | | | | | | |
| 13 | **Model 3:** | | | | | **Score:** | **Rank** |
| 14 | Amy | *0.3* | *0.3* | *0.2* | *0.1* | 0.88 | 1 |
| 15 | Betty | 0.22 | 0.27 | 0.20 | 0.15 | 0.83 | 2 |
| 16 | Cindy | 0.28 | 0.27 | 0.14 | 0.05 | 0.74 | 5 |
| 17 | Daisy | 0.22 | 0.23 | 0.19 | 0.10 | 0.74 | 6 |
| 18 | Emily | 0.13 | 0.27 | 0.20 | 0.05 | 0.65 | 8 |
| 19 | Fanny | 0.17 | 0.30 | 0.16 | 0.05 | 0.68 | 7 |
| 20 | Ginny | 0.23 | 0.20 | 0.17 | 0.15 | 0.75 | 4 |
| 21 | Hillary | 0.27 | 0.20 | 0.12 | 0.20 | 0.79 | 3 |

$$\text{Final score} = \frac{p_1 A}{A_{\max}} + \frac{p_2 B}{B_{\max}} + \frac{p_3 C}{C_{\max}} + \frac{p_4 D}{D_{\max}}$$

## Communicate

A clear report on the model, solution technique, and results makes the model understandable to others

Padlet

## Model 1 and Model 2

| | Annual average score | Interview | Attendance | Languages | Mean (Model 1) | |
|---|---|---|---|---|---|---|
| wight *(w)* | *0.4* | *0.3* | *0.2* | *0.1* | | **Rank** |
| A | 4 | 4 | 4 | 2 | 3.50 | 1 |
| B | 1 | 3 | 4 | 3 | 2.75 | 2 |
| C | 3 | 3 | 2 | 1 | 2.25 | 6 |
| D | 1 | 2 | 4 | 2 | 2.25 | 6 |
| E | 1 | 3 | 4 | 1 | 2.25 | 6 |
| F | 1 | 4 | 3 | 2 | 2.50 | 3 |
| G | 2 | 1 | 3 | 3 | 2.25 | 6 |
| H | 3 | 1 | 1 | 4 | 2.25 | 6 |

| **Model 2:** | | | | | **Score:** | **Rank** |
|---|---|---|---|---|---|---|
| Amy | 1.6 | 1.2 | 0.8 | 0.2 | 3.8 | 1 |
| Betty | 0.4 | 0.9 | 0.8 | 0.3 | 2.4 | 4 |
| Cindy | 1.2 | 0.9 | 0.4 | 0.1 | 2.6 | 2 |
| Daisy | 0.4 | 0.6 | 0.8 | 0.2 | 2.0 | 8 |
| Emily | 0.4 | 0.9 | 0.8 | 0.1 | 2.2 | 5 |
| Fanny | 0.4 | 1.2 | 0.6 | 0.2 | 2.4 | 3 |
| Ginny | 0.8 | 0.3 | 0.6 | 0.3 | 2.0 | 8 |
| Hillary | 1.2 | 0.3 | 0.2 | 0.4 | 2.1 | 6 |

$$\text{Final score} = \frac{w_1x_1 + w_2x_2 + w_3x_3 + w_4x_4}{w_1 + w_2 + w_3 + w_4}$$

## Model 3

| | Annual average score | Interview | Attendance | Languages |
|---|---|---|---|---|
| proportions *(p)* | *0.4* | *0.3* | *0.2* | *0.1* |
| A | 90 | 9 | 90 | 2 |
| B | 65 | 8 | 100 | 3 |
| C | 85 | 8 | 70 | 1 |
| D | 65 | 7 | 95 | 2 |
| E | 40 | 8 | 100 | 1 |
| F | 50 | 9 | 80 | 2 |
| G | 70 | 6 | 85 | 3 |
| H | 80 | 6 | 60 | 4 |
| max | **90** | **9** | **100** | **4** |

| **Model 3:** | | | | | **Score:** | **Rank** |
|---|---|---|---|---|---|---|
| Amy | 0.40 | 0.30 | 0.18 | 0.05 | 0.93 | 1 |
| Betty | 0.29 | 0.27 | 0.20 | 0.08 | 0.83 | 2 |
| Cindy | 0.38 | 0.27 | 0.14 | 0.03 | 0.81 | 3 |
| Daisy | 0.29 | 0.23 | 0.19 | 0.05 | 0.76 | 5 |
| Emily | 0.18 | 0.27 | 0.20 | 0.03 | 0.67 | 8 |
| Fanny | 0.22 | 0.30 | 0.16 | 0.05 | 0.73 | 7 |
| Ginny | 0.31 | 0.20 | 0.17 | 0.08 | 0.76 | 6 |
| Hillary | 0.36 | 0.20 | 0.12 | 0.10 | 0.78 | 4 |

$$\text{Final score} = \frac{p_1A}{A_{max}} + \frac{p_2B}{B_{max}} + \frac{p_3C}{C_{max}} + \frac{p_4D}{D_{max}}$$

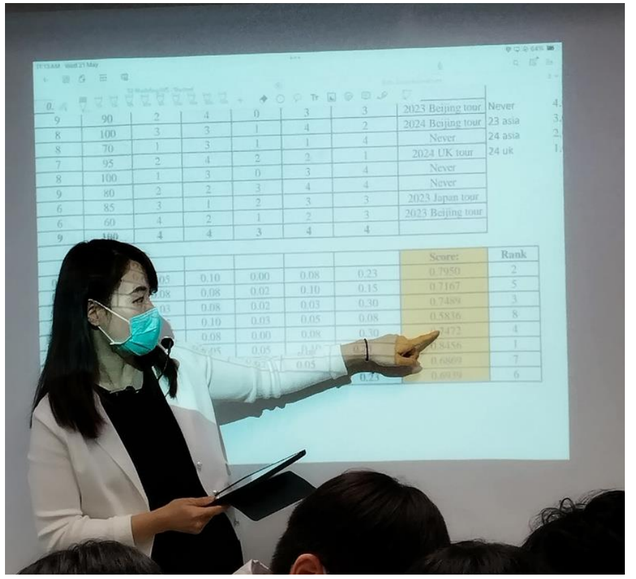
## Model 3 further

| | Annual average score | Interview | Attendance | Languages | Conduct | Posts | Feedback | Year Joined tour before | Year Joined tour before | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| proportions *(p)* | *0.10* | *0.20* | *0.05* | *0.10* | 0.10 | 0.05 | 0.10 | 0.30 | | | |
| Amy | 90 | 9 | 90 | 2 | 4 | 0 | 3 | 3 | 2023 Beijing tour | Never | 4.00 |
| Betty | 65 | 8 | 100 | 3 | 3 | 1 | 4 | 2 | 2024 Beijing tour | 23 asia | 3.00 |
| Cindy | 85 | 8 | 70 | 1 | 3 | 1 | 1 | 4 | Never | 24 asia | 2.00 |
| Daisy | 65 | 7 | 95 | 2 | 4 | 2 | 2 | 1 | 2024 UK tour | 24 uk | 1.00 |
| Emily | 40 | 8 | 100 | 1 | 3 | 0 | 3 | 4 | Never | | |
| Fanny | 50 | 9 | 80 | 2 | 2 | 3 | 4 | 4 | Never | | |
| Ginny | 70 | 6 | 85 | 3 | 1 | 2 | 3 | 3 | 2023 Japan tour | | |
| Hillary | 80 | 6 | 60 | 4 | 2 | 1 | 2 | 3 | 2023 Beijing tour | | |
| **max** | **90** | **9** | **100** | **4** | **4** | **3** | **4** | **4** | | | |

| **Model 3:** | | | | | | | | | **Score:** | **Rank** |
|---|---|---|---|---|---|---|---|---|---|---|
| Amy | 0.10 | 0.20 | 0.0450 | 0.05 | 0.10 | 0.00 | 0.08 | 0.23 | 0.7950 | 2 |
| Betty | 0.07 | 0.18 | 0.0500 | 0.08 | 0.08 | 0.02 | 0.10 | 0.15 | 0.7167 | 5 |
| Cindy | 0.09 | 0.18 | 0.0350 | 0.03 | 0.08 | 0.02 | 0.03 | 0.30 | 0.7489 | 3 |
| Daisy | 0.07 | 0.16 | 0.0475 | 0.05 | 0.10 | 0.03 | 0.05 | 0.08 | 0.5836 | 8 |
| Emily | 0.04 | 0.18 | 0.0500 | 0.03 | 0.08 | 0.00 | 0.08 | 0.30 | 0.7472 | 4 |
| Fanny | 0.06 | 0.20 | 0.0400 | 0.05 | 0.05 | 0.05 | 0.10 | 0.30 | 0.8456 | 1 |
| Ginny | 0.08 | 0.13 | 0.0425 | 0.08 | 0.03 | 0.03 | 0.08 | 0.23 | 0.6869 | 7 |
| Hillary | 0.09 | 0.13 | 0.0300 | 0.10 | 0.05 | 0.02 | 0.05 | 0.23 | 0.6939 | 6 |

## Data Sheet (Part 2)

| **Students** | **Criteria** | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Annual result last year (0 – 100) | Interview performance (0 – 10) | Attendance rate in % (0 – 100) | Number of spoken Languages (1 – 4) | Conduct (A – D) | Number of service posts (0 – 5) | Feedback from teachers (Poor, Fair, Good, Excellent) | Joined tour before? Which year? Which tour? |
| **Amy** | 90 | 9 | 90 | 2 | A | 0 | Good | 2023 Beijing tour |
| **Betty** | 65 | 8 | 100 | 3 | B | 1 | Excellent | 2024 Beijing tour |
| **Cindy** | 85 | 8 | 70 | 1 | B | 1 | Poor | Never |
| **Daisy** | 65 | 7 | 95 | 2 | A | 2 | Fair | 2024 UK tour |
| **Emily** | 40 | 8 | 100 | 1 | B | 0 | Good | Never |
| **Fanny** | 50 | 9 | 80 | 2 | C | 3 | Excellent | Never |
| **Ginny** | 70 | 6 | 85 | 3 | D | 2 | Good | 2023 Japan tour |
| **Hillary** | 80 | 6 | 60 | 4 | C | 1 | Fair | 2024 Beijing tour |

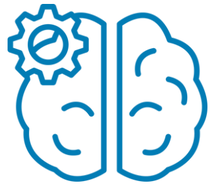


# What does the math modeling process look like?

## Define the Problem

Modeling problems are open-ended

Define with their 4 points scale for each group



## Make Assumptions

It is impossible to account for all the factors that impact a given situation,

so we must choose what is most important criteria to include in the study tour.

## Define Variables

Making assumptions reveals variables that will be needed.

Each group defines each weights for each criteria. Weights become variables among different groups.

## Get a Solution

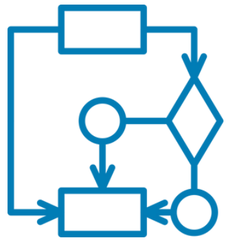
Use familiar mathematical and computational tools to solve..

Using google sheet/ excel as tools to calculate rapidly.

# What does the math modeling process look like?

## Analyze the Solution

When considering the results and insights gained from the model, ask how to improve from the model.
Why is the different results from different group?



## Iterate

*Refine the model by repeating the process, adjusting as needed to improve the solution.*
Instead of repeating the model, we compare the different results with the same model among different groups to save time.

## Communicate

A clear report on the model, solution technique, and results makes the model understandable to others

Reference: https://m3challenge.siam.org/what-is-math-modeling/

# S4 Mathematical Modelling Project Presentation

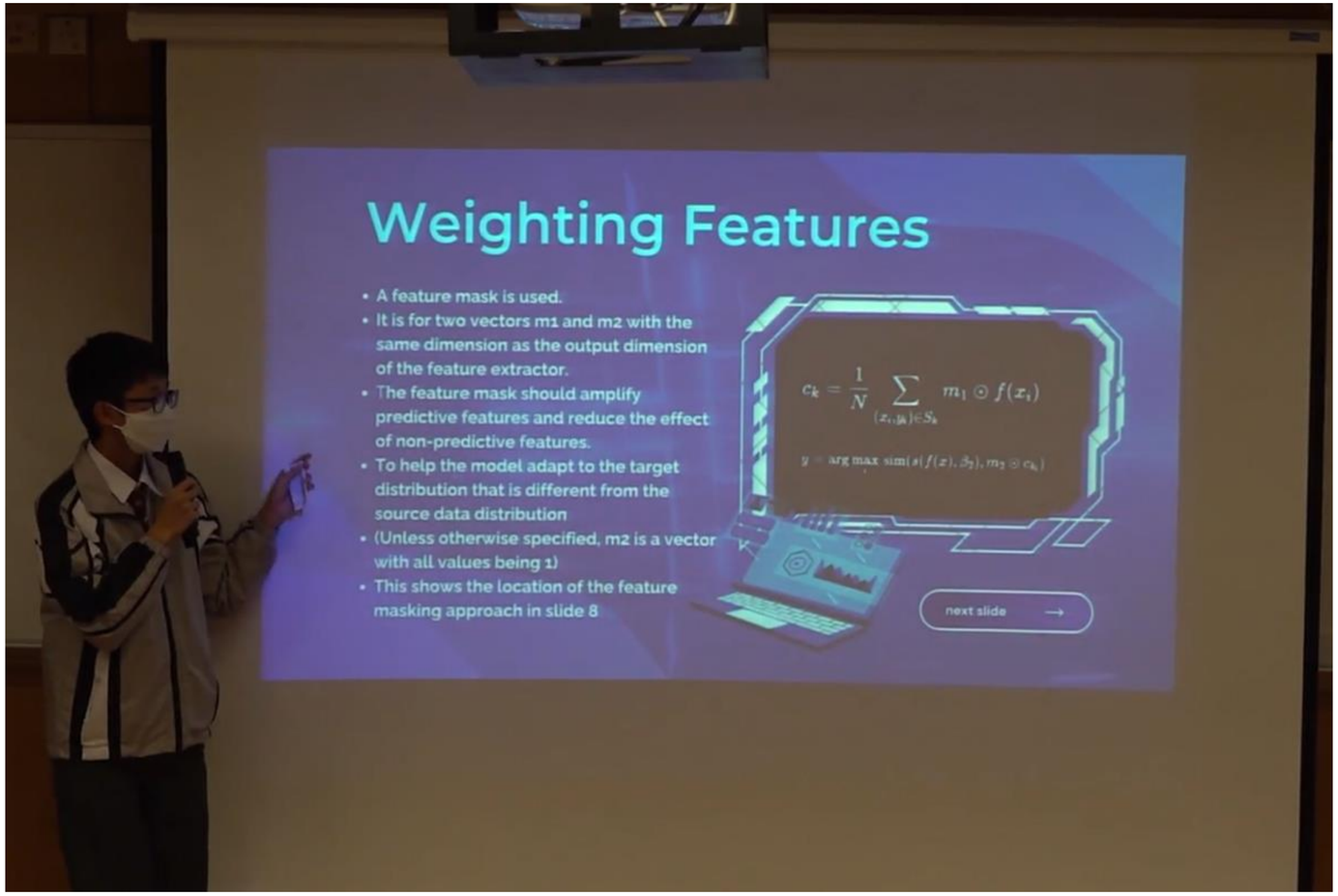
**Date : 21 March 2025**

**Time : 14 : 20 – 15 : 40**

**Class: S.4 (Class M2)**

**Subject Teacher:**
**Mr. Tommy Chiang**

# Group 1
# Prototypical Networks with Feature Masking

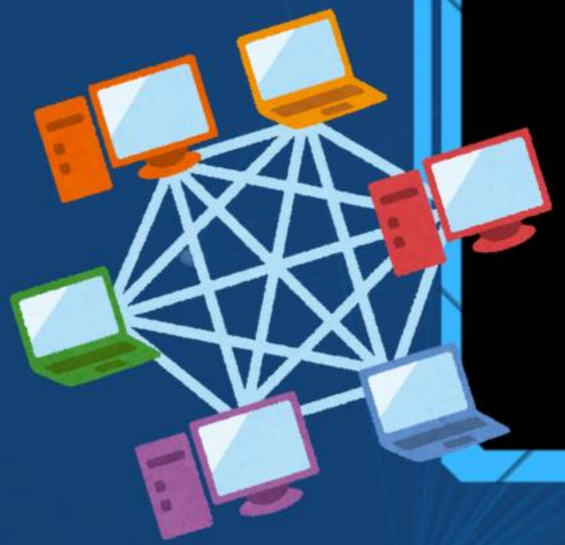


# 1. Class Prototype Calculation

The class prototype $c_k$ for class $k$ is calculated as:

$$c_k = \frac{1}{N} \sum_{(x_i, y_i) \in S_k} f(x_i)$$

- **Variables:**
  - $c_k$: Class prototype for class $k$.
  - $S_k$: Support set containing samples of class $k$.
  - $N$: Number of samples in the support set $S_k$.
  - $f(x_i)$: Feature extractor function applied to input $x_i$.



next slide →

| ViT-B-14 Results | | | | | |
|---|---|---|---|---|---|
| | Aircraft | Flowers | CIFAR | CUB | mini-ImageNet |
| Baseline | 74.13 | 99.94 | 96.29 | 98.82 | 97.48 |
| Exponent (k=0.3) | 74.28 | 99.94 | 96.66 | 98.79 | 97.97 |
| Exponent (k=0.5) | 74.21 | 99.94 | **96.76** | 98.76 | **98.18** |
| Exponent (k=2) | 73.79 | 99.94 | 95.80 | 98.57 | 98.14 |
| Boolean mask ($\gamma$=0.1) | 74.56 | 99.94 | 96.00 | 98.81 | 97.36 |
| Boolean mask ($\gamma$=0.2) | **74.89** | 99.94 | 95.67 | **98.83** | 97.22 |

| ViT-S-14 Results | | | | | |
|---|---|---|---|---|---|
| | Aircraft | Flowers | CIFAR | CUB | mini-ImageNet |
| Baseline | 74.21 | **99.90** | 94.86 | 98.12 | 97.42 |
| Exponent (k=0.3) | 74.72 | 99.89 | **94.89** | 98.10 | 97.73 |
| Exponent (k=0.5) | 74.65 | 99.89 | 94.82 | 98.06 | **97.81** |
| Exponent (k=2) | 73.86 | 99.87 | 92.87 | 97.61 | 97.12 |
| Boolean mask ($\gamma$=0.1) | 74.96 | 99.89 | 94.30 | **98.13** | 96.79 |
| Boolean mask ($\gamma$=0.2) | **75.43** | **99.90** | 93.41 | 98.03 | 97.22 |

Math-group 2

# modelling on Livability of a Country/Region

4A 19, 24, 27, 29
4C 18

# Part 04

# Entropy weight method

# Calculation Steps of Entropy Weight Method

Normalization of Data

Mapping the data to the [0,1] range for ease of subsequent calculations.

Entropy Value Calculation

$$H_i = -k \sum_{j=1}^{m} p_{ij} \ln p_{ij}$$

determination of the weight

$$w_i = \frac{1 - H_i}{\sum_{j=1}^{n} (1 - H_j)}$$

## 4. Constructing the Livability Index

We define the ***Livability Index (L)*** as a weighted sum of multiple sub-indices, each representing a critical aspect of livability.

$$L = \sum_{i=1}^{n} w_i I_i$$

where:

- $I_i$ is the value of the ***i-th livability factor*** (scaled appropriately).
- $W_i$ is the **weight of the factor**, determined using **entropy weighting**.

# Example Calculation With Real Datas: step 1 collect data

| Indicator | GDP per capita (USD) | Median Income (USD) | Gini Coefficient | Crime Rate (per 100k) | PM2.5 (μg/m³) | Renewable Energy (%) | Green Space (%) | Education Index (0-1) | Healthcare Quality Index | Cultural Diversity Index | Political Stability | Rule of Law Index |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Hong Kong,China | 49800 | 24000 | 0.539 | 60 | 25 | 1 | 40 | 0.91 | 85 | 0.75 | 0.2 | 0.65 |
| USA | 76400 | 45000 | 0.415 | 380 | 8 | 21 | 34 | 0.89 | 80 | 0.82 | 0.5 | 0.73 |
| Germany | 54100 | 37000 | 0.316 | 200 | 12 | 46 | 55 | 0.92 | 87 | 0.78 | 1.1 | 0.81 |
| Japan | 40800 | 33000 | 0.329 | 35 | 18 | 24 | 68 | 0.88 | 84 | 0.72 | 0.8 | 0.79 |
| Brazil | 8900 | 4500 | 0.539 | 2200 | 45 | 85 | 60 | 0.75 | 65 | 0.89 | -1.2 | 0.46 |

# Step 2 Calculate Sub-Indices for Hong Kong

**Economic Index (IE)**

$$I_E = \frac{\text{GDP} + \text{Median Income} - (\text{Gini} \times 100)}{\max(\text{GDP}) + \max(\text{Income})} = \frac{49,800 + 24,000 - 53.9}{76,400 + 45,000} = 0.607$$

**Security Index (IS)**

$$I_S = \frac{\text{Normalized Crime Score} + \text{Political Stability (0–100)} + \text{Rule of Law (0–100)}}{3} = \frac{97.27 + 54 + 65}{3} = 72.09 \quad \text{(Scaled to 0.721)}$$

**Environment Index (IEnv)**

$$I_{\text{Env}} = \frac{100 - 25 + 1 + 40}{100} = 1.16 \quad \text{(Capped at 1.0)}$$

**Social Index (IC)**

$$I_C = \frac{0.91 + 0.85 + 0.75}{3} = 0.837$$

# Step 3 Calculate Entropy for each Sub-Indices

| Region | IE | IS | IEnv | IC |
|---|---|---|---|---|
| Hong Kong | 0.607 | 0.721 | 1.0 | 0 837 |
| USA | 0.792 | 0.623 | 0.98 | 0.823 |
| Germany | 0.665 | 0.689 | 1.0 | 0.857 |
| Japan | 0.502 | 0.812 | 0.93 | 0.813 |
| Brazil | 0.0 | 0.402 | 0.56 | 0.633 |

For IE

$$H_E = -\frac{1}{\ln(5)} \sum p_{ij} \ln p_{ij} = 0.914$$

Similarly, HS= 0.892, HEnv=0.963, HC= 0.901

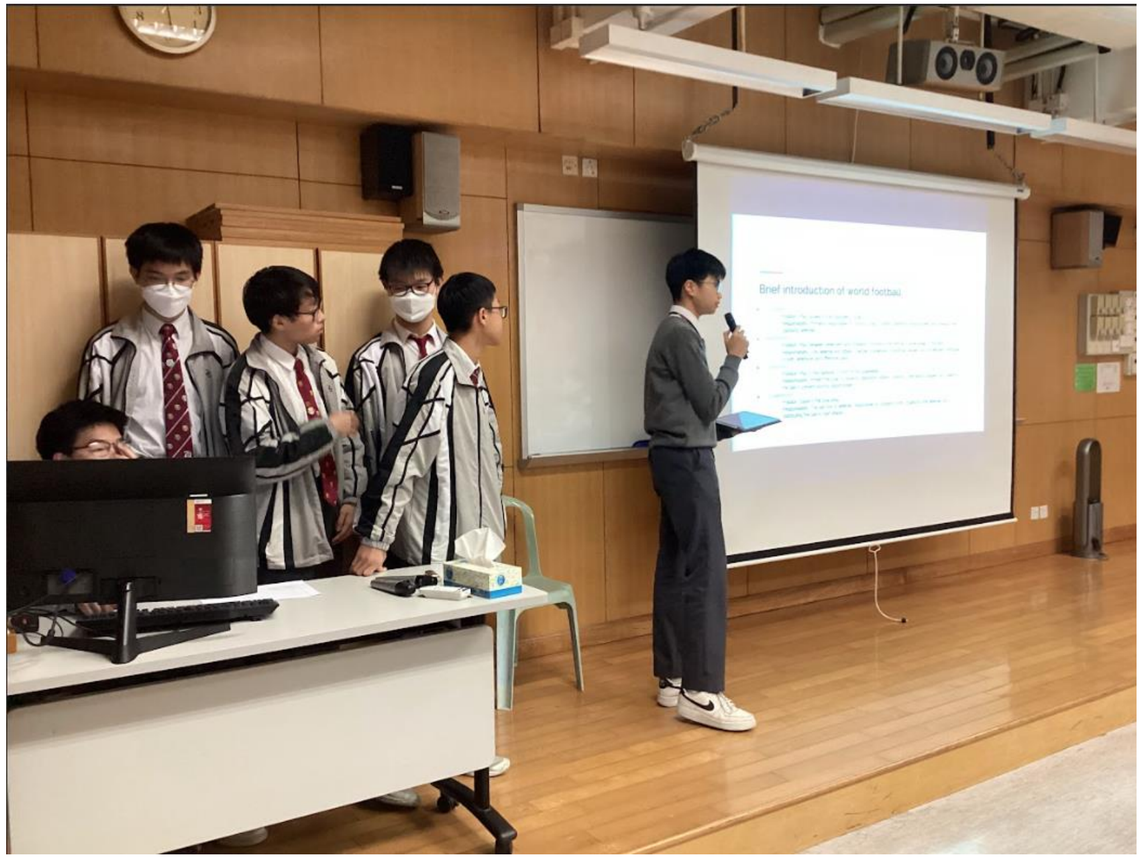
# Global Case Studies of Typical Regions

| Country/Region | GDP/Capita | Crime Rate | PM2.5 | Education | Final L |
|---|---|---|---|---|---|
| Hong Kong | 49,800 | 60 | 25 | 0.91 | 0.75 |
| USA | 76,400 | 380 | 8 | 0.89 | 0.78 |
| Germany | 54,100 | 200 | 12 | 0.92 | 0.80 |
| Japan | 40,800 | 35 | 18 | 0.88 | 0.73 |
| Brazil | 8,900 | 2,200 | 45 | 0.75 | 0.48 |

North America Region Case Study-USA

Asian Region Case Study-Japan

# Group 3
# The Best Footballer in the world



# Normalizing the scoring criterias

In order to avoid large numbers which may be difficult to manage, we have normalized each of the scoring criterias to a common scale. For each scoring criteria of a player X , the normalized value X*norm* is calculated as:

$$X_{norm} = \frac{X - X_{min}}{X_{max} - X_{min}}$$

Where Xmin = the minimum record of a same position player in that criteria
Xmax = the maximum record of a same position player in that criteria

# Step 9: Comparing the weighted sum of the best player in different positions

Goalkeeper: Mark Flekken (0.81)
Defender: Antonee Robinson (0.53)
Midfielder: Bruno Fernandes (0.61)
Forward: Mohamed Salah (0.75)

# Limitations of the Modelling

1. As the weights are made up and decided by our team, it is subjective and different people may have different views about the importance of each quantity to the position. Therefore, the result generated by different people's weighting may be different.
2. As it is hard to find some indicators measuring those intangible qualities (e.g., leadership, teamwork) , we can't use a mathematical way to estimate the ability of players including these qualities.
3. As we used only a few scoring criteria, the result generated may not be so accurate as the criteria cannot fully reflect the ability of the players.
4. As the players are playing for different clubs, they have different teammates and face different opponents, so it is hard for us to give a true and fair rating for all the players. For example, when a stronger team face a weaker team, the goalkeeper and defenders of the stronger team may have less pressure and less chances of receiving the ball or to make an interception compared to facing a equally strong team, vice versa.

# Mathematical Modelling Project

## Economy Index of a Region / Country

(Group 4)

# Modelling Methods

**1. Entropy Weight Method:**
To obtain the weightings of each indicator

2. **TOSIS**:
To generate the factor score

# Data Preprocessing

rix P:

| 2022 | Unemployment R | GDP per capita ( | Inflation Rate(%) | Average Salary(USD) |
|---|---|---|---|---|
| Kong | 0.10 | 0.22 | 0.24 | 0.18 |
| nited State | 0.12 | 0.24 | 0.07 | 0.25 | |
| | 0.06 | 0.01 | 0.07 | 0.01 |
| | 0.10 | 0.04 | 0.04 | 0.03 |
| ny | 0.15 | 0.19 | 0.06 | 0.20 |
| | 0.17 | 0.15 | 0.15 | 0.17 |
| | 0.13 | 0.04 | 0.06 | 0.03 |
| co | 0.04 | 0.01 | 0.07 | 0.02 |
| Arabia | 0.07 | 0.08 | 0.18 | 0.08 |
| | 0.06 | 0.02 | 0.05 | 0.02 |
| | | | | |
| Check) | 1.00 | 1.00 | 1.00 | 1.00 |

## 01.

**Standardisation**

$$\frac{1}{x}$$

$$X = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1m} \\ x_{21} & x_{22} & \cdots & x_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nm} \end{bmatrix}$$

## 02.

**Normalisation**

$$z_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{n} x_{ij}^2}}$$

$$\tilde{Z} = \begin{bmatrix} \tilde{z}_{11} & \tilde{z}_{12} & \cdots & \tilde{z}_{1m} \\ \tilde{z}_{21} & \tilde{z}_{22} & \cdots & \tilde{z}_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{z}_{n1} & \tilde{z}_{n2} & \cdots & \tilde{z}_{nm} \end{bmatrix}$$

## 03.

**Forming The Weighting Matrix**

$$p_{ij} = \frac{\tilde{z}_{ij}}{\sum_{i=1}^{n} \tilde{z}_{ij}}$$

$$\sum_{i=1}^{n} p_{ij} = 1.$$

# Entropy Weight Method: Basic Idea - Entropy

**Entropy is the scale of the average level of uncertainty or information**

**-The level of uncertainty ↑ (Probability ↓)**

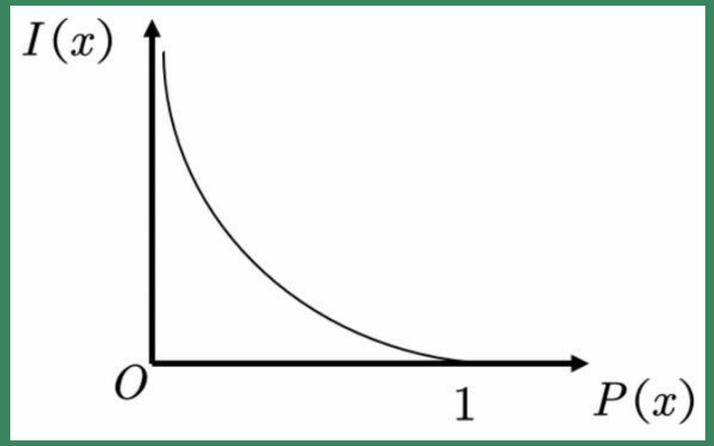
**-The amount of information can be derived ↑**

**-The entropy ↑**

**-The reliability of that indicators ↓**

**-The weighting ↓**

$$I(x) = -\ln(p(x))$$



# Entropy Weight Method

$$e_j = -\frac{1}{\ln n}\sum_{i=1}^{n} p_{ij}\ln(p_{ij})\ (j=1,2,\cdots,m)$$

$$d_j = 1 - e_j$$

$$W_j = d_j / \sum_{j=1}^{m} d_j\ (j=1,2,\cdots,m)$$

| A | B | C | D | E | F |
|---|---|---|---|---|---|
| (j=1, 2, 3…) | Unemployment R | GDP per capita ( | Inflation Rate(%) | Average Salary(USD) | |
| wj (weighting of | 0.07638881768 | 0.3783957553 | 0.1738680537 | 0.3713473733 | |
| Sum of wj (Chec | 1 | | | | |

# TOPSIS

Normalised Data:

| 2022 | Unemployment R | GDP per capita ( | Inflation Rate(%) | Average Salary(USD) |
|---|---|---|---|---|
| Hong Kong | 0.2938 | 0.5302 | 0.6416 | 0.4262 |
| The United State | 0.3509 | 0.5740 | 0.1875 | 0.5964 |
| India | 0.1779 | 0.0175 | 0.1820 | 0.0321 |
| Russia | 0.3081 | 0.0925 | 0.1024 | 0.0785 |
| Germany | 0.4355 | 0.4426 | 0.1543 | 0.4821 |
| Japan | 0.4858 | 0.3690 | 0.4063 | 0.4177 |
| Mexico | 0.3827 | 0.0850 | 0.1563 | 0.0601 |
| Morocco | 0.1128 | 0.0283 | 0.1935 | 0.0404 |
| Saudi Arabia | 0.2178 | 0.1927 | 0.4876 | 0.1994 |
| Peru | 0.1779 | 0.0580 | 0.1451 | 0.0590 |
| | | | | |
| Matrix Z+ (Max. V | 0.4858 | 0.5740 | 0.6416 | 0.5964 |
| Matrix Z- (Min. V | 0.1128 | 0.0175 | 0.1024 | 0.0321 |
| | | | | |
| Weightings (wj) | 0.07638881768 | 0.3783957553 | 0.1738680537 | 0.3713473733 |

$$Z^+ = (Z_1^+, Z_2^+, \cdots, Z_m^+) = (\max\{z_{11}, z_{21}, \cdots, z_{n1}\}, \max\{z_{12}, z_{22}, \cdots, z_{n2}\}, \cdots, \max\{z_{1m}, z_{2m}, \cdots, z_{nm}\})$$

$$Z^- = (Z_1^-, Z_2^-, \cdots, Z_m^-) = (\min\{z_{11}, z_{21}, \cdots, z_{n1}\}, \min\{z_{12}, z_{22}, \cdots, z_{n2}\}, \cdots, \min\{z_{1m}, z_{2m}, \cdots, z_{nm}\})$$

| 2022 | Unemployment R | GDP per capita ( | Inflation Rate(%) | Average Salary(U | D+ |
|---|---|---|---|---|---|
| Hong Kong | 0.0028 | 0.0007 | 0.0000 | 0.0108 | 0.1195789801 |
| The United State | 0.0014 | 0.0000 | 0.0359 | 0.0000 | 0.1929841993 |
| India | 0.0072 | 0.1172 | 0.0367 | 0.1182 | 0.5285871856 |
| Russia | 0.0024 | 0.0877 | 0.0505 | 0.0996 | 0.4901969335 |
| Germany | 0.0002 | 0.0065 | 0.0413 | 0.0049 | 0.2299239721 |
| Japan | 0.0000 | 0.0159 | 0.0096 | 0.0119 | 0.1933570992 |
| Mexico | 0.0008 | 0.0905 | 0.0409 | 0.1068 | 0.4889265309 |
| Morocco | 0.0106 | 0.1127 | 0.0349 | 0.1148 | 0.5225114699 |
| Saudi Arabia | 0.0055 | 0.0550 | 0.0041 | 0.0585 | 0.3509309546 |
| Peru | 0.0072 | 0.1008 | 0.0429 | 0.1072 | 0.5080327256 |

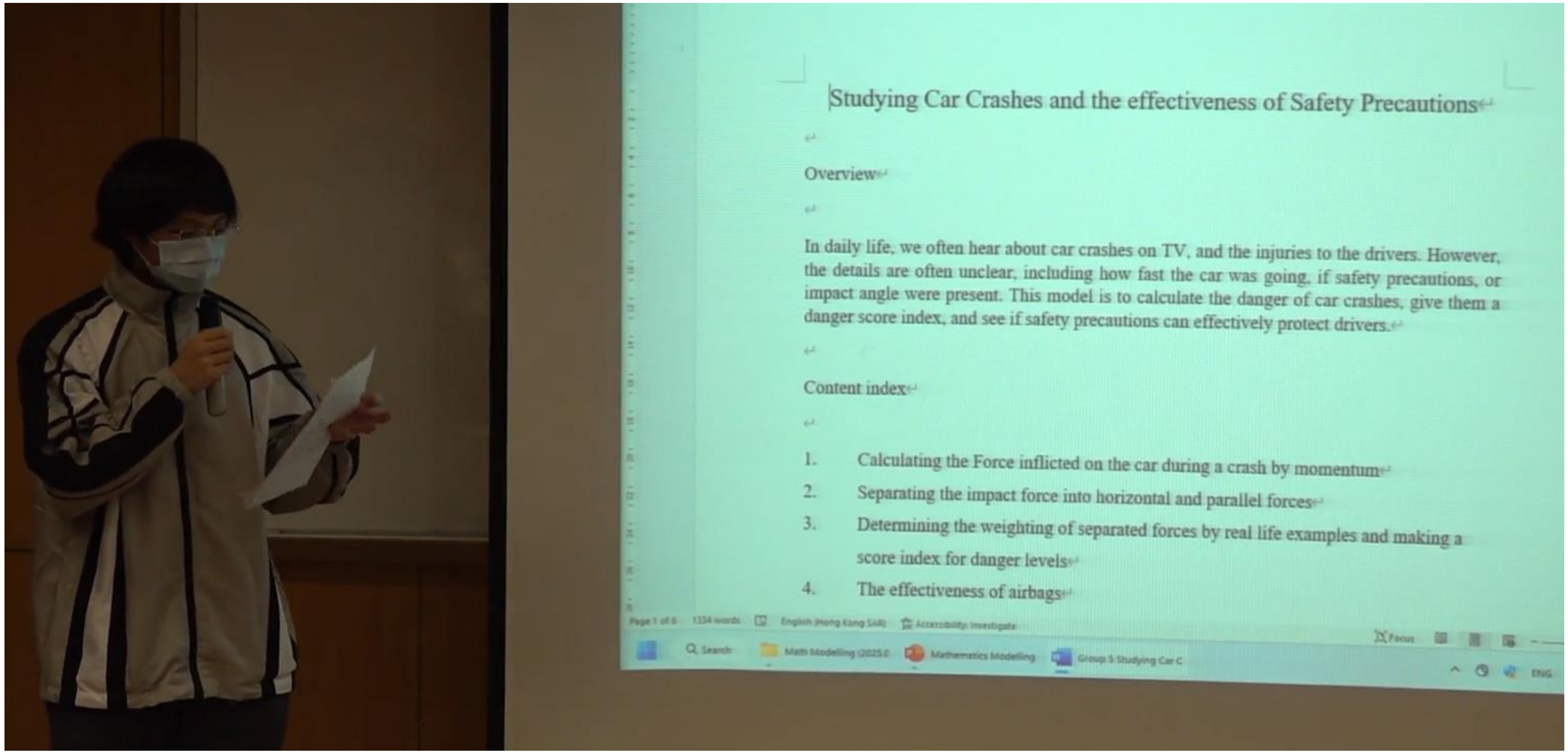
$$D_i^+ = \sqrt{\sum_{j=1}^{m} \omega_j (Z_j^+ - z_{ij})^2}$$

$$D_i^- = \sqrt{\sum_{j=1}^{m} \omega_j (Z_j^- - z_{ij})^2}$$

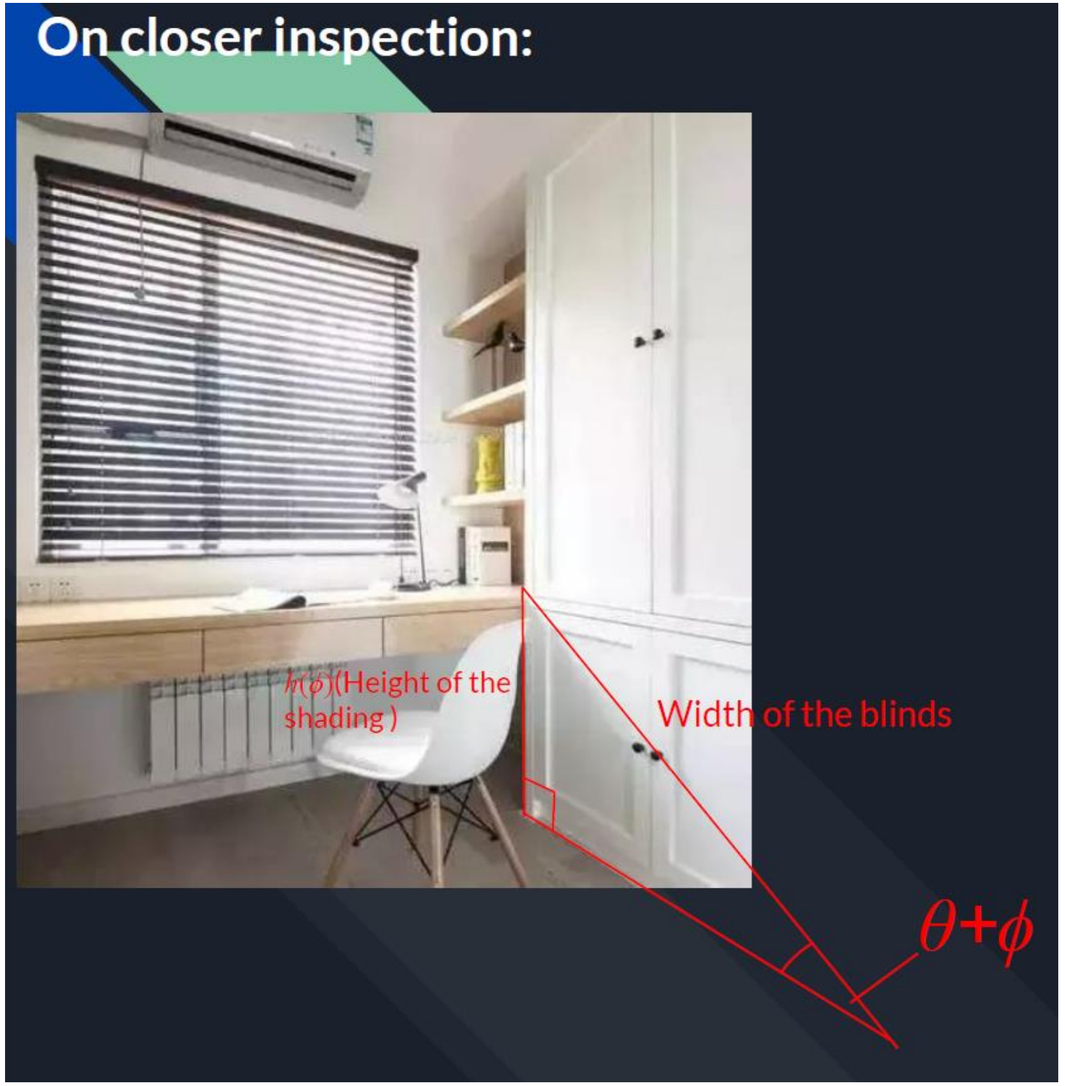
| 2022 | Unemployment R | GDP per capita ( | Inflation Rate(%) | Average Salary(U | D- | Economic Index | Rank |
|---|---|---|---|---|---|---|---|
| Hong Kong | 0.0025 | 0.0995 | 0.0505 | 0.0577 | 0.4584688985 | 0.793 | 1 |
| The United State | 0.0043 | 0.1172 | 0.0013 | 0.1182 | 0.4909437886 | 0.718 | 2 |
| India | 0.0003 | 0.0000 | 0.0011 | 0.0000 | 0.03775436902 | 0.067 | 10 |
| Russia | 0.0029 | 0.0021 | 0.0000 | 0.0008 | 0.07643036912 | 0.135 | 7 |
| Germany | 0.0080 | 0.0684 | 0.0005 | 0.0752 | 0.3898728616 | 0.629 | 4 |
| Japan | 0.0106 | 0.0468 | 0.0161 | 0.0552 | 0.3586806769 | 0.650 | 3 |
| Mexico | 0.0056 | 0.0017 | 0.0005 | 0.0003 | 0.08991632658 | 0.155 | 6 |
| Morocco | 0.0000 | 0.0000 | 0.0014 | 0.0000 | 0.03889326023 | 0.069 | 9 |
| Saudi Arabia | 0.0008 | 0.0116 | 0.0258 | 0.0104 | 0.2205655997 | 0.386 | 5 |
| Peru | 0.0003 | 0.0006 | 0.0003 | 0.0003 | 0.0391167837 | 0.071 | 8 |

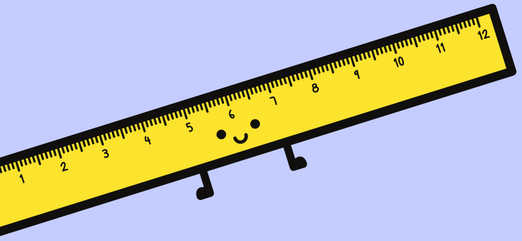
$$S_i = \frac{D_i^-}{D_i^+ + D_i^-}$$

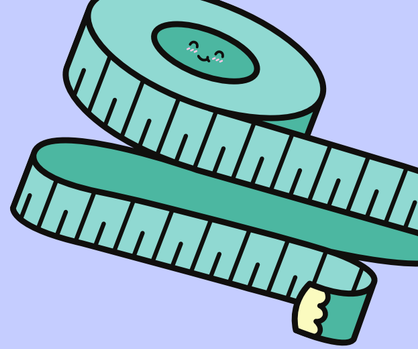
# Group 5
# Studying Car Crashes and the effectiveness of Safety Precautions

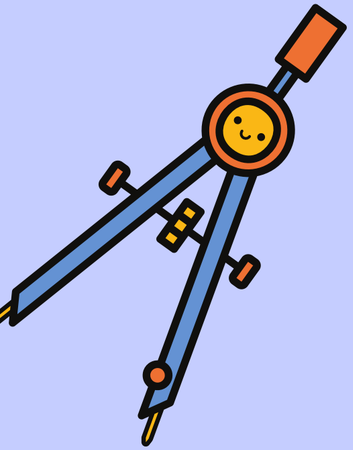


# Group 6
# Optimal Blind Angle Model for Sunlight Shading

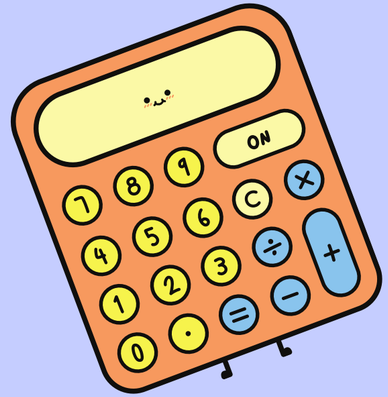


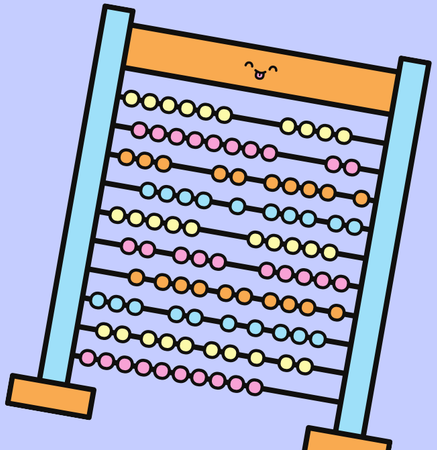




# 參與學校分享

可立中學（嗇色園主辦）

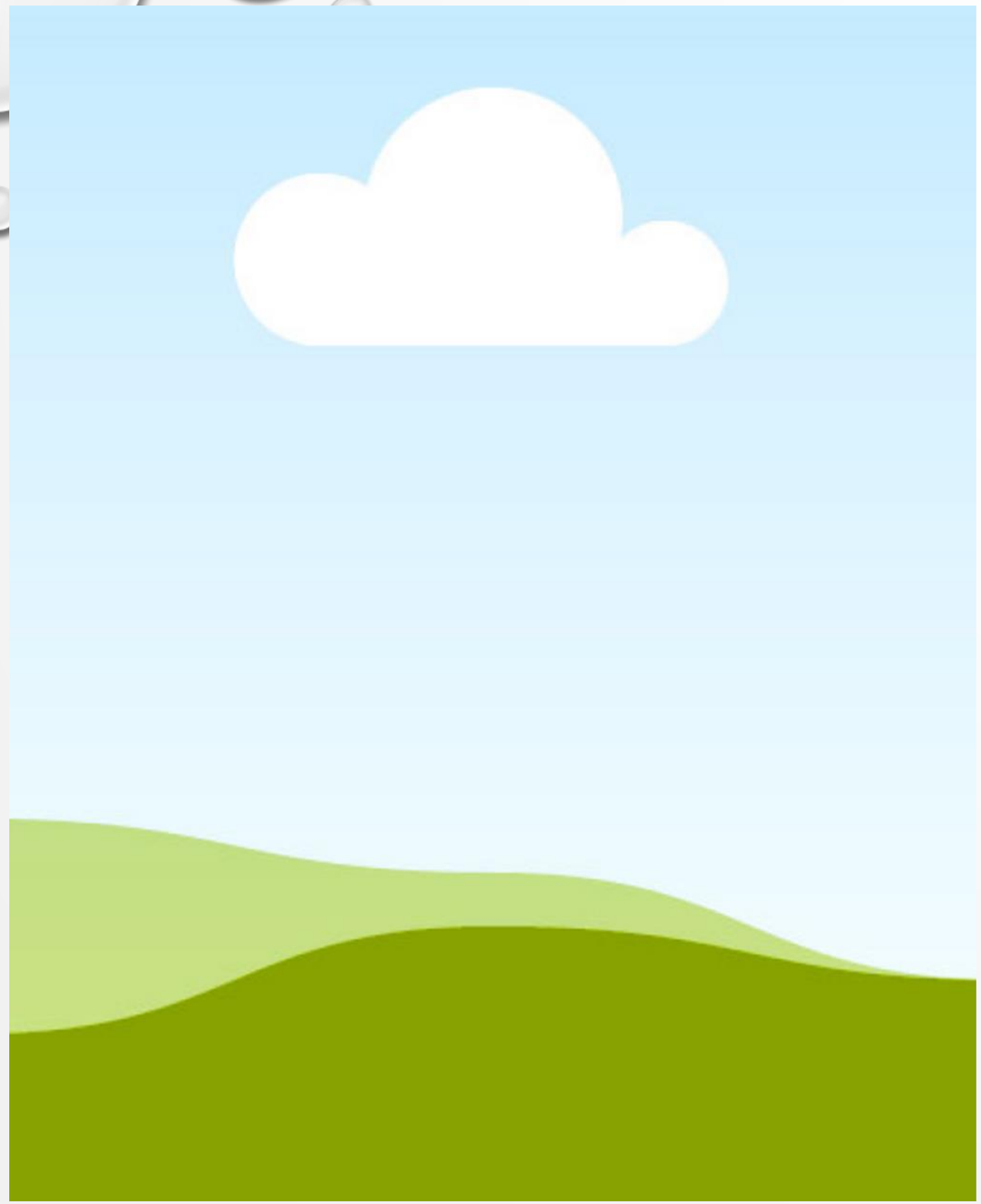




# QEF Thematic Network on "Promotion of STEAM Education by Infusing Mathematical Modelling into Secondary Mathematics" (2024/25)

## 將數學建模注入中學數學科以推展STEAM教育(2024/25)

**Ho Lap College (Sponsored by Sik Sik Yuen)**



# Why joining this QTN Project?

- Good mathematical skills in general.
- Students are focusing the examination-type questions only.
- Seek more opportunities for them to discuss, create and present their ideas in Mathematical Lesson.
- Nurture the professional development of mathematics teachers to guide students to understand, learn and use mathematical modelling to solve problems.

# Schedule

First term:

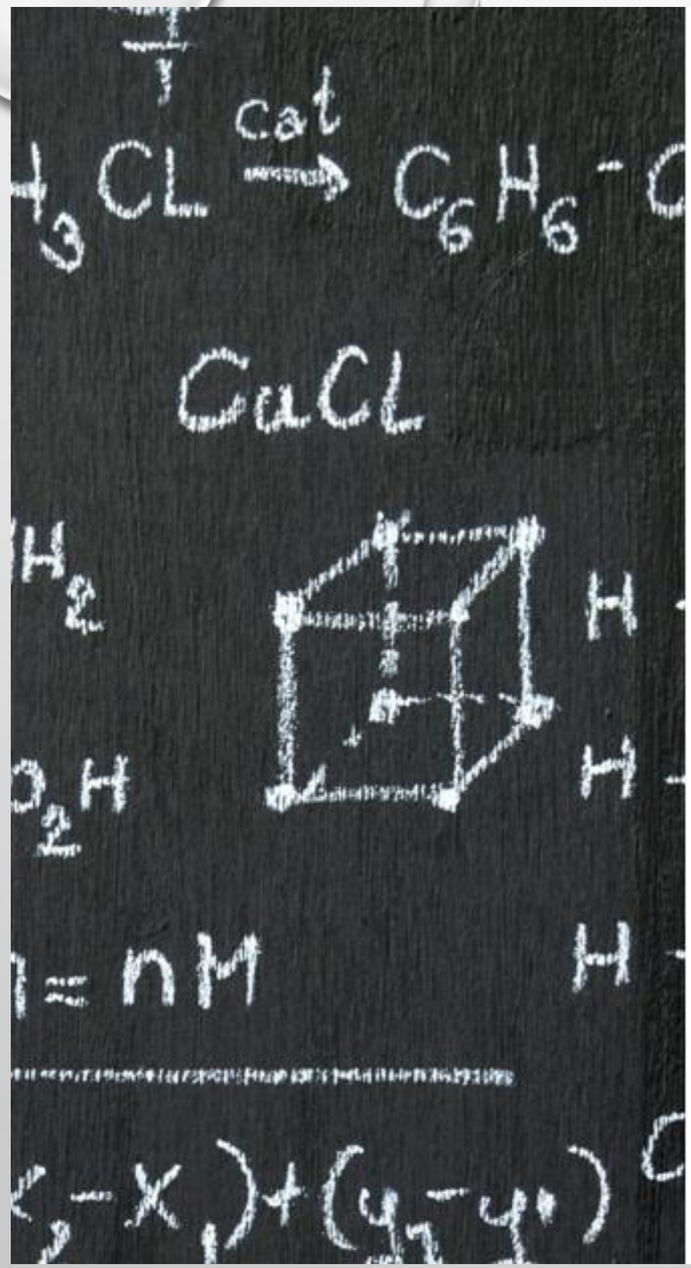
- Two S4 classes with middle to high abilities

Second term

- Tasting lessons for two S3 classes with middle to high abilities
- Follow-up work for first term activity for S4 classes


# First term activity

- Two S4 classes
- Venue: Makerspace (STEAM room)
- Work in Group




# Mathematical Modelling

- Mathematical modelling is the process of describing a real world problem in mathematical terms, usually in the form of equations, and then using these equations both to help understand the original problem, and also to discover new features about the problem.

# Mathematical Modelling Cycle

| | |
|---|---|
| Problem | Identify and observe relations between two or more variables. |
| Formulate | Guess which model/relation may be suitable |
| Compute | Collect data and compute suitable model to have “Best –Fit” |
| Interpret | Explain why the relation happens and other factors may affect. |
| Validate | Collect more data to check your model |
| Report | Report and explain how your model can fit and predict the data. |

# Real-life Problem

Based on Transport Department of Hong Kong, we have the following data.

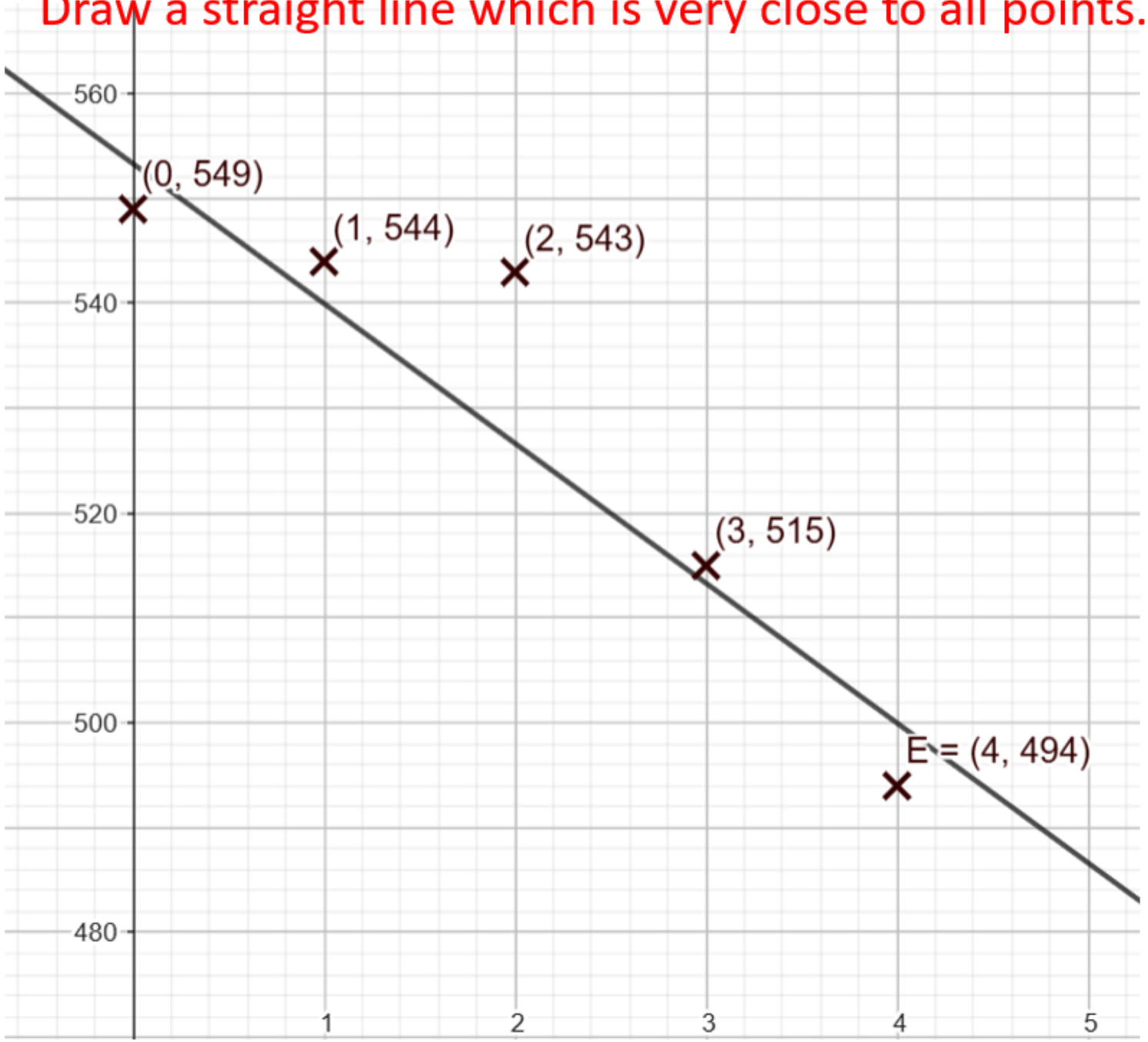
| Year | 2019 | 2020 | 2021 | 2022 | 2023 |
|---|---|---|---|---|---|
| Total Licensed Petrol Cars (in 1000) | **549** | **544** | **543** | **515** | **494** |

# Real-life Problem

In order to have easier calculation, treat x as the following

| Year | 2019 | 2020 | 2021 | 2022 | 2023 |
|---|---|---|---|---|---|
| x | 0 | 1 | 2 | 3 | 4 |
| Total Licensed Petrol Cars (in 1000) | 549 | 544 | 543 | 515 | 494 |

Draw a straight line which is very close to all points.



# Activity I

Application of Mathematical skills Our students have learnt

## Introduction

## Task 1: Linear Model

The set of data below follows a linear model $y = ax + b$, where $a$ and $b$ are constants.

| $x$ | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| $y$ | 5 | 9 | 13 | 17 | 21 |

(a) Find the constants $a$ and $b$ and hence an equation connecting $x$ and $y$ .

(b) Find $y$ when $x = 11$ .

## Task 2: Quadratic Model

The set of data below follows a quadratic model $y = ax^2 + bx + c$, where $a$ and $b$ are constants.

| $x$ | 1 | 2 | 3 | 4 | 6 |
|---|---|---|---|---|---|
| $y$ | 3 | 2 | 3 | 6 | 18 |

(a) Find the constants $a$, $b$ and $c$ and hence an equation connecting $x$ and $y$ .

(b) Find $y$ when $x = 7.5$ .

## ratic Model

a below follows a quadratic model $y = ax^2 + bx + c$, where

| $x$ | 1 | 2 | 3 | 4 | 6 |
|---|---|---|---|---|---|
| $y$ | 3 | 2 | 3 | 6 | 18 |

the constants $a$, $b$ and $c$ and hence an equation connecting $x$ and $y$.

$y$ when $x = 7.5$.

$3 = a + b + c$

$3 - b - c$

$2 = 4a + 2b + c$

$2 = 12 - 4b - 4c + 2b + c$

$-10 = -2b - 3c$

$c = \frac{-10 + 2b}{-3}$

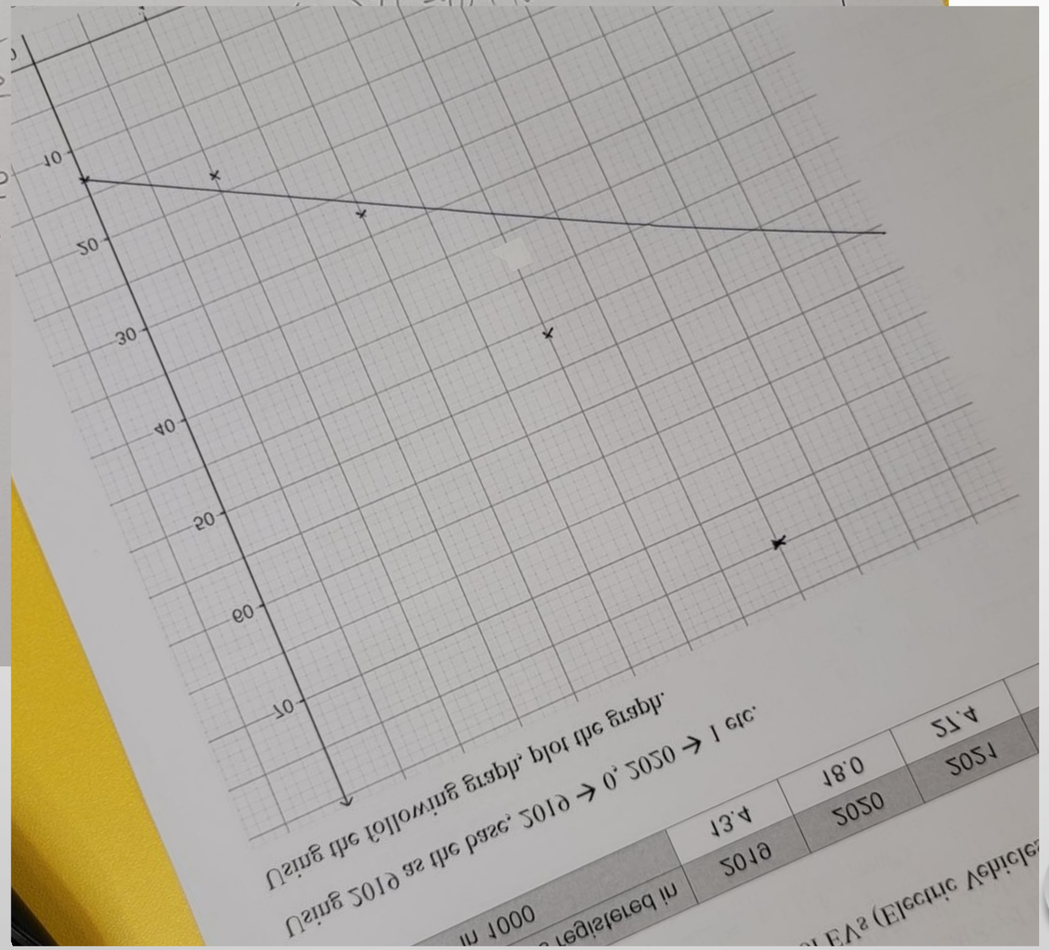
$c = \frac{10 - 2b}{3}$

$3 = 9a + 3b + c$

$3 = 9(3 - b - c) + 3b + \left(\frac{10 - 2b}{3}\right)$

$3 = 27 - 9b - 9\left(\frac{10 - 2b}{3}\right) + 3b + \frac{10 - 2b}{3}$

$-90 + 18b + 108 + 10 - 2b$

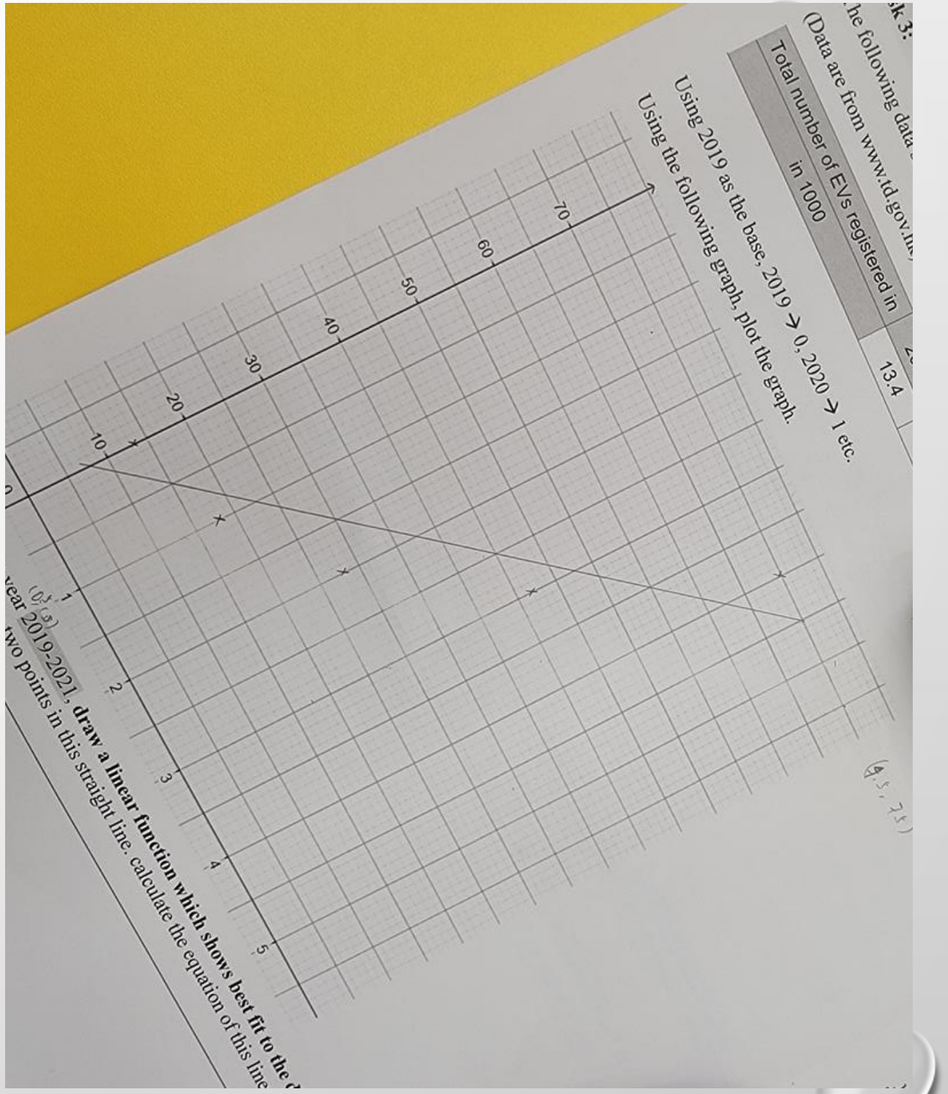


Using the following graph, plot the graph.

Using 2019 as the base, 2019 → 0, 2020 → 1 etc.

| Total number of EVs registered in 1000 | 13.4 |
|---|---|

(Data are from www.id.gov.in



Using 2019 as the base, 2019 → 0, 2020 → 1 etc.

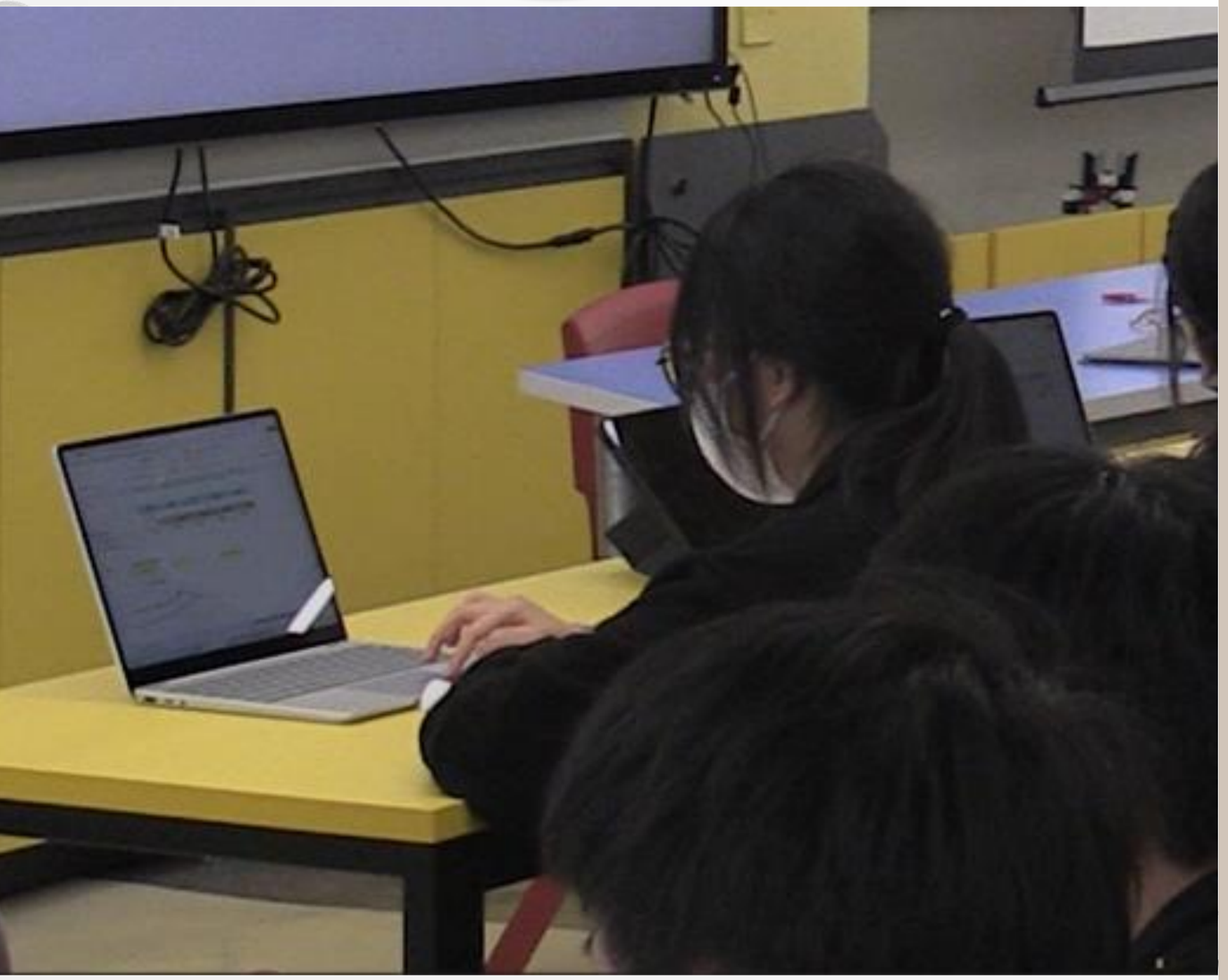
Using the following graph, plot the graph.

| Total number of EVs registered in 1000 | 13.4 |
|---|---|

(Data are from www.id.gov.in

**ear 2019-2021, draw a linear function which shows best fit to the**

**two points in this straight line. calculate the equation of this line**



# Activity II

Using excel to let students discuss and try different model by trial and error

**Group Number:** _____

**Topic of Your Group:** _____

**Group Members:**

| Class | Class No. | Name | Class | Class No. | Name |
|---|---|---|---|---|---|
| | | | | | |
| | | | | | |
| | | | | | |

**Your Data**

| | 2019 | 2020 | 2021 | 2022 | 2023 |
|---|---|---|---|---|---|
| x | 0 | 1 | 2 | 3 | 4 |
| Real | | | | | |
| y | | | | | |
| Error/Residual | | | | | |

*Please submit the following results in google form provided.*

Your estimated model: Linear/ Quadratic

| | |
|---|---|
| $y = mx + k$ | $m =$<br>$k =$ |
| $y = ax^2 + bx + c$ | $a =$<br>$b =$<br>$c =$ |

**Discussion**

1. Why does your group choose this model to estimate the model?
2. Search the information from the Internet, discuss the trends of data and corresponding factors affecting.

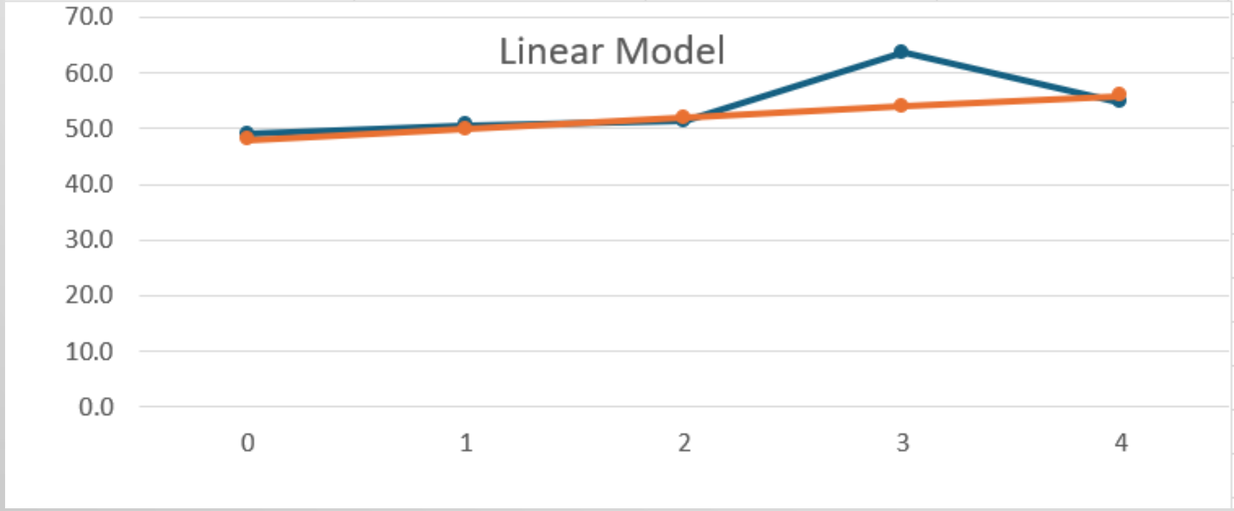
Group Work
Using Excel to explore the data
Test and Try and feedback in Google Form

Title

| | | 2019 | 2020 | 2021 | 2022 | 2023 |
|---|---|---|---|---|---|---|
| x | x | 0 | 1 | 2 | 3 | 4 |
| Real Figures | | **49.0** | **50.7** | **51.4** | **63.7** | **54.7** |
| Estimated Value | y | **48.0** | **50.0** | **52.0** | **54.0** | **56.0** |
| Error, Residual | | 1.0 | 0.3 | 2.6 | 6.7 | 5.3 |

Model: y=mx+k

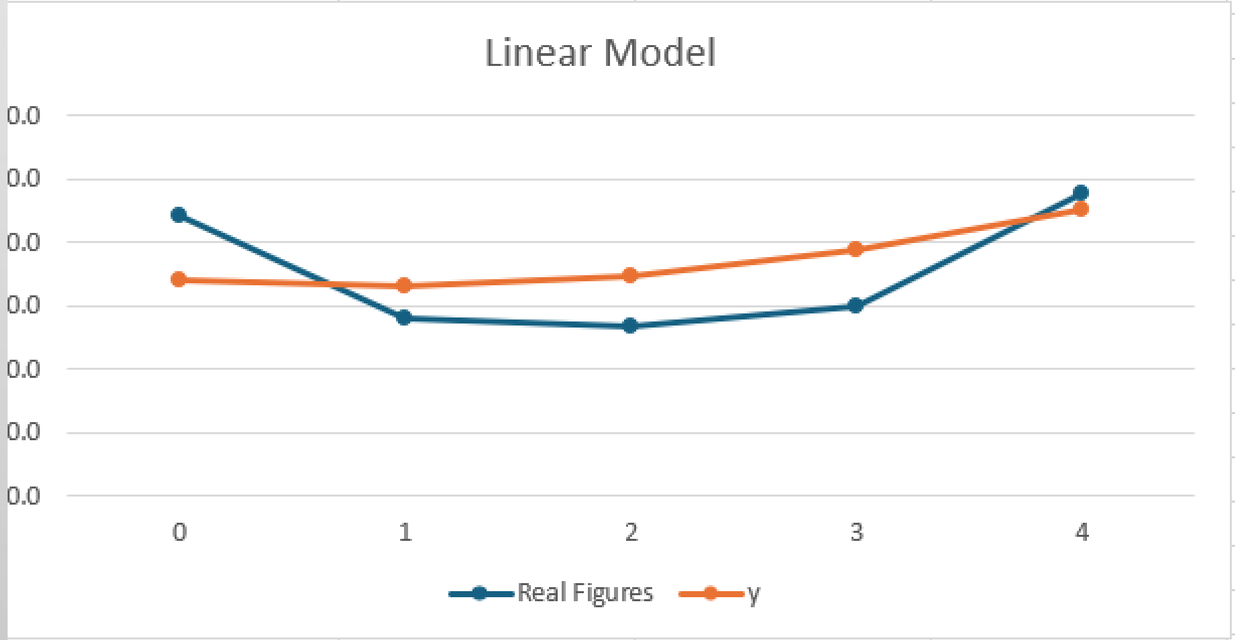
n= 2 k= 48



| | | 2019 | 2020 | 2021 | 2022 | 2023 |
|---|---|---|---|---|---|---|
| x | x | 0 | 1 | 2 | 3 | 4 |
| eal Figures | | 44.2 | 27.9 | 26.9 | 30.0 | 47.7 |
| stimated Value | y | 34.0 | 33.2 | 34.8 | 38.8 | 45.2 |
| rror, Residual | | | | | | |

odel: $y=ax^2+bx+c$

a= 1.2 b= -2 c= 34



Timestamp

| | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| 1 | Form_Responses1 | | | | | | | |
| 2 | Timestamp | Class | Group | m= | c= | a= | b= | c= |
| 3 | 13/12/2024 12:07:05 | 4D | Group 3 | 4 | -3 | 1 | -4 | |
| 4 | 13/12/2024 12:09:36 | 4D | Group 4 | 4 | -3 | 1 | -4 | |
| 5 | 13/12/2024 12:14:36 | 4D | Group 3 | 4 | -3 | 1 | -4 | |
| 6 | 13/12/2024 12:15:14 | 4C | Group 4 | 4 | -3 | 1 | -4 | |
| 7 | 13/12/2024 12:16:54 | 4C | Group 4 | 4 | -3 | 1 | -4 | |
| 8 | 13/12/2024 12:19:24 | 4D | Group 1 | 4 | -3 | 1 | -4 | |
| 9 | 13/12/2024 18:07:36 | 4C | Group 3 | 4 | -3 | 1 | -4 | |

| | m= | c= | a= | b= | c= |
|---|---|---|---|---|---|
| 6 | 5 | 13 | 2.4 | 2.2 | 13.4 |
| 6 | 7 | 13.4 | 0 | 7 | 13.4 |
| 6 | 5 | 13 | 2.5 | 2.5 | 13 |
| 6 | 6.3 | 13.4 | 2.4 | 2.2 | 13.4 |
| 6 | 4.5 | 8 | 2.4 | 2.2 | 13.4 |
| 6 | 13.75 | 10 | 2.4 | 2.2 | 13.4 |
| 6 | 7 | 13.4 | 0.62 | 17.38 | -4.54 |

# Reflection

Time control

Lack of time for letting students present their work

Need more discussion time

Post-lesson time for ss to design what data they like to explore more

# Second term activity

Two S3 classes

Introduction to the concepts of mathematical model by probability

# Objectives

Using the concepts and probabilities, discuss with students how to win the prize

Discuss and let students explore difficult situations.

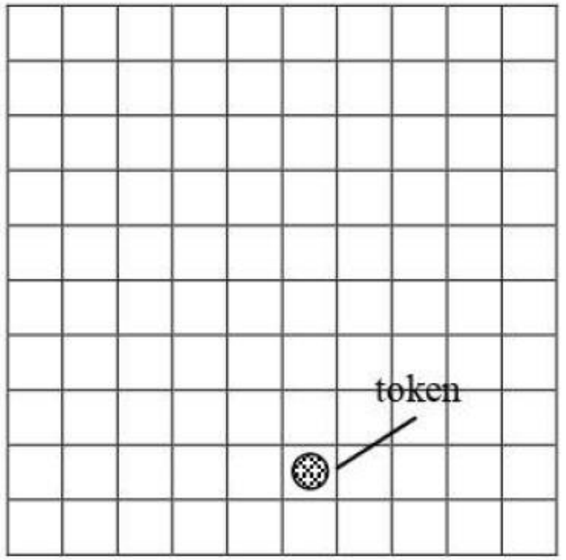
**Objective:** (1) To enrich students' experience in applying mathematics in handling daily life problems

(2) To enhance students' abilities in applying probability in modelling real-life situations

Have you ever played a game in which you throw a token at random onto a tiled floor with tiles of the same size in order to win a prize?

**Activity 1:**

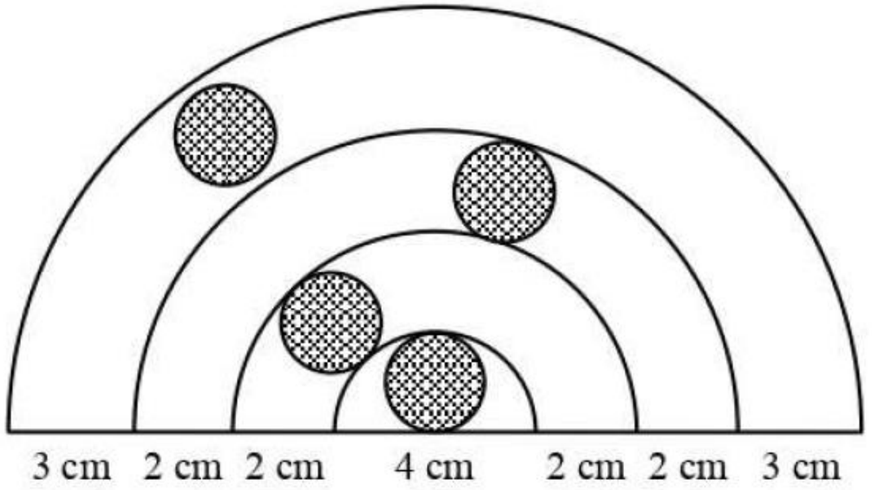
A square floor is tiled with 100 identical square tiles. A player throws a token onto the floor at random and she/he will win a prize if the token lands entirely within the boundaries of a single tile.

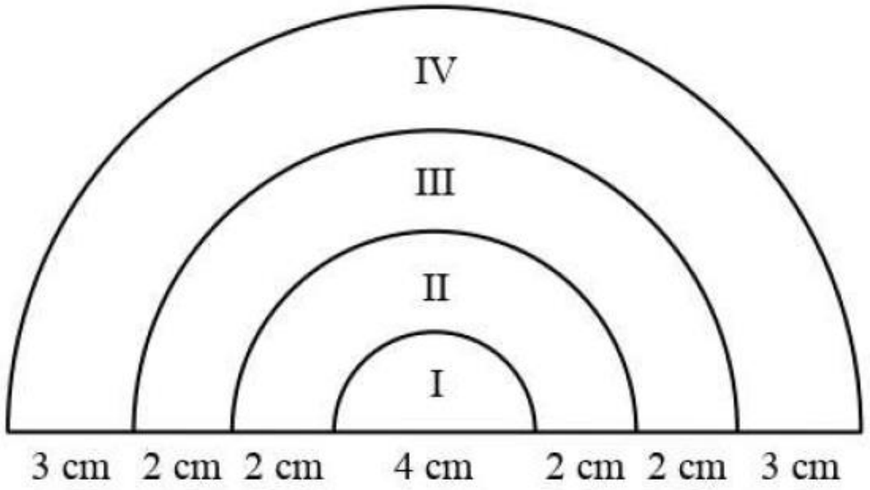



It is given that the size of each tile is 3.2 cm × 3.2 cm and the diameter of a token is 2 cm .

**Activity 2:**

The game below models the game in Jumpin Gym HLC. The diameter of a token is again 2 cm .




If the token is located inside region IV , there will be a small prize. If the token is located is located in region III , there will be a medium prize. If the token is located is located in region II , there will be a large prize. If the token is located is located in region I , there will be a special prize.

(a) What is the probability of getting a small prize?


We will be using
the writing in
Eng lesson. Keep
it with you
Highlight:
- conditionals
- Adj. pattern
HOMEWORK RECORD

# Reflection

Lack of time to let students play the game first

Next year, integrate with steam lesson to design the board by students

# 參與學校分享

香港中文大學校友會聯會張煊昌中學

# 數學建模之實踐

香港中文大學校友會聯會張煊昌中學
CUHK FAA Thomas Cheung Secondary School
黃智傑老師
趙潤老師
李健朗老師
蕭錦泓老師

# 為什麼做數學建模？

數學不只算式，是解決現實問題的工具

# 課堂案例一：如何分配溫習時間

- 📄 任務說明
- 學生任務：在 5 天內、共 20小時自習時間中，分配給 3科考試復習用
- 包括：中文、英文、數學（每科重要程度與熟練度不同）
- 學生須根據條件，設計出「最合理」的時間分配方案
- 📦 建模元素
- 條件限制：總時間 = 20 小時；每科至少需安排一點時間
- 數據處理：表格紀錄各科重要性與熟練度；計算需求分數
- 模型設計：需求分數 = 重要性 ÷ 熟練度
- 決策分析：按比例分配時間

3. **公式的設計邏輯：**
    - 分配時間的核心是**「需求」**，需求越高，時間分配越多。
    - 每科的需求可以用以下方式表示：

$$\text{需求分數} = \frac{\text{重要性分數}}{\text{熟練程度}}$$

    - **重要性高（分子大）：** 增加需求，時間多一些。
    - **熟練程度高（分母大）：** 減少需求，時間少一些。

- **總時間分配：**
    - 將每科需求分數計算出來，然後按比例分配總時間 T。
    - 公式： $\text{分配時間} = T \cdot \frac{\text{需求分數}}{\text{總需求}}$

# 課堂案例二：設計最划算的校外教學行程

- ✏️ 任務說明
- 學生任務：在預算 $200 和時間 6 小時內，規劃一日行程
- 包括：交通、景點、午餐、活動（選填）
- 每個選項有費用、時間和滿意度
- 🧱 建模元素
- 條件限制：費用 ≦ $200；總時間 ≦ 6 小時
- 數據處理：表格 / 計算總費用總時間 / 平均或加權滿意度
- 決策分析：比較組合、找出最划算方案

⏱ 總時間計算

| 項目 | 時間（小時） |
| --- | --- |
| 交通 | |
| 景點 1 | |
| 景點 2 | |
| 午餐 | |
| 活動 | |
| **總計** | **_____ 小時 /6 小時** ✅ / ❌ |

⭐ 滿意度分析

請將所選項目的滿意度加總或平均。

☑ 我們選擇的計算方式是：

☐ 加總　　☐ 平均值

# 我們怎樣引導學生建模？

- 💡 有意識地引導學生經歷建模五步驟，用提問激發思考與修正

| 建模步驟 | 教師提問語句 | 學生可能反應 |
| --- | --- | --- |
| 1. 理解問題 | 「你覺得這個問題要解決什麼？」<br>「目標是什麼？」 | 「要規劃一日行程，不能超過預算和時間。」 |
| 2. 整理資訊 | 「你需要哪些資料才能做決定？」<br>「資訊從哪裡來？」 | 「費用、時間、滿意度……都在表格裡。」 |
| 3. 建立數學模型 | 「我們可以用什麼方法來整理這些資訊？」<br>「怎樣比較組合？」 | 「用表格列出各項資料，計算總費用和滿意度。」 |
| 4. 分析與計算 | 「這個方案可行嗎？」<br>「有沒有超過時間或預算？」 | 「這樣超時了，我們要換掉一個景點或交通方式。 |
| 5. 解釋與修正模型 | 「如果條件改變了，你會怎麼調整？」<br>「有沒有更好的方案？」 | 「如果預算變少，我會改搭地鐵＋便當節省開支。」 |

# 我們的收穫與反思

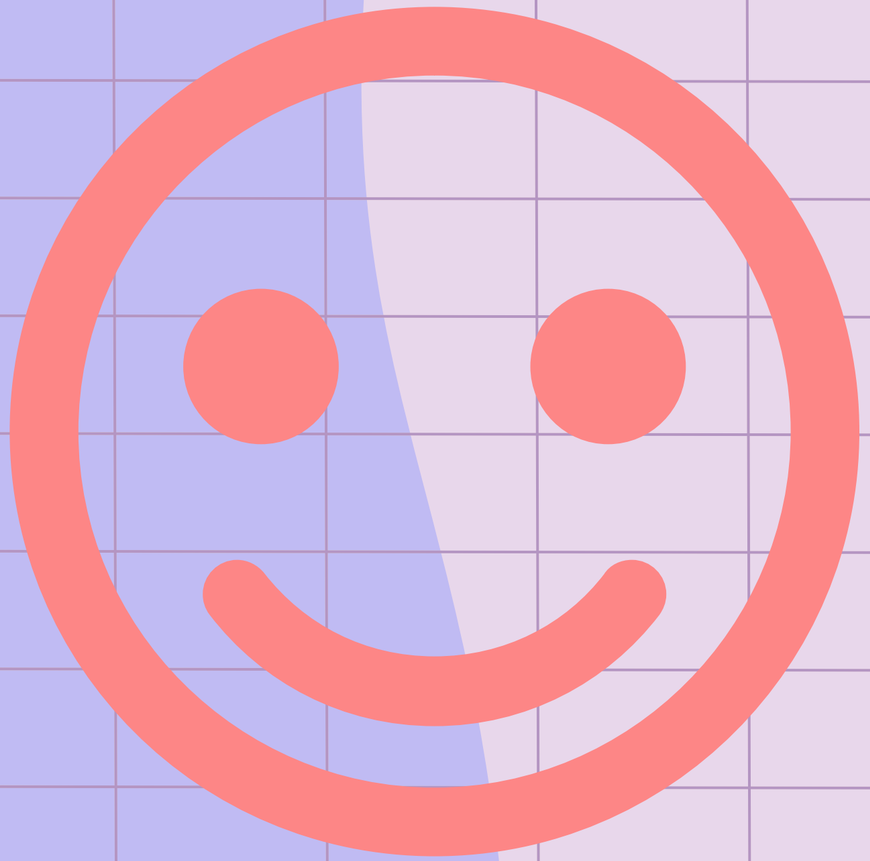
| 學生的學習轉變 ✨ | 教師的觀察與反思 👀 |
| --- | --- |
| **從算式操作 → 解決問題**<br>不再只是「代公式」，而是「要解決什麼問題？」 | 我們看到學生會主動分析條件、選擇策略，不再只問「對不對？」 |
| **從答題思維 → 決策思維**<br>開始比較方案、評估選擇的優劣 | 學生說：「這和我們生活中真的很像！」參與度明顯提升 |
| **從一次完成 → 願意修正與優化**<br>會因限制條件調整模型或方案 | 學生願意修正錯誤、討論更好的方法，展現出建模的精神 |

# 我們還在學習，但想分享三個小心得：

| 心得 | 我們的實踐方式 |
| --- | --- |
| 1. 從生活出發 | 選學生熟悉的題材，如旅行、零用錢、時間分配 |
| 2. 加一點限制 | 用預算、時間、條件，讓問題變得「真實又值得想」 |
| 3. 放手讓學生選擇 | 不急著給答案，讓學生比較、討論、修正自己的想法 |

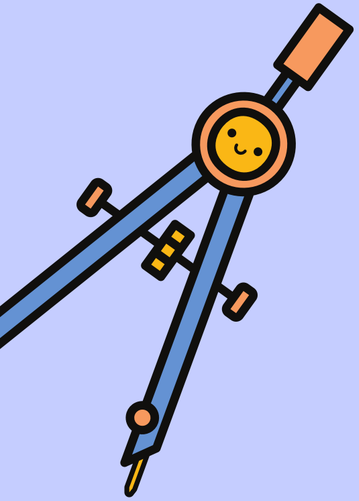
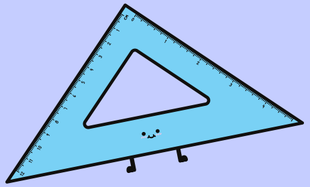
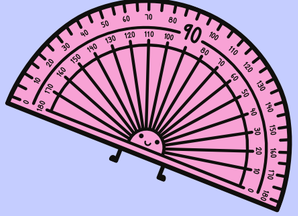
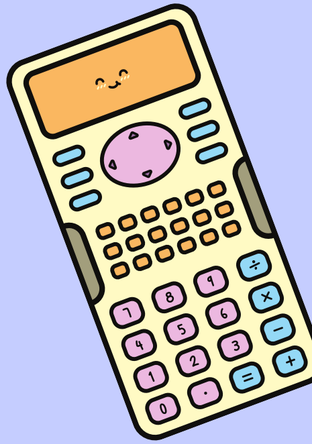
**我們還在摸索，但真的覺得：**
**從生活問題 + 設限制條件 + 放手讓學生選擇，**
**建模精神就自然發生了！**

# Thank You



# 總結

- 各校從數學建模帶給學生與別不同的學習模式
- 期待各校因應校情繼續發展數學建模
- **Train-the-trainer**：下學年各校嘗試建立自己的網絡

# Thank you!!!

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