Overview of PISA

The Programme for International Student Assessment (PISA) is a project initiated and coordinated by the Organisation of Economic Co-operation and Development (OECD). The primary goal of this international study is to assess how well youths at age 15, near the completion of compulsory education, have acquired the knowledge and skills essential for meeting the challenges of our society. It then develops educational indicators to help governmental bodies and policy makers examine, evaluate, and monitor the effectiveness of the educational system at both national and school levels.

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The Assessment of Mathematical Literacy
Learning from PISA

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【PISA Assessment Framework and Pedagogical Practice Series】

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PISA Assessment Framework and Pedagogical Practice Series

Since 2000, the Hong Kong Centre for International Student Assessment (HKPISA Centre) had conducted four cycles of the OECD Programme for International Student Assessment (PISA). Three major domains of literacy are assessed in PISA: reading, mathematics and science, as well as the literacy of problem solving which was assessed in PISA 2003.

While the PISA assessment frameworks are constructed by international experts based on sound theories, the test items are developed following the framework closely. The PISA frameworks and test items would be valuable references for teachers. It is hoped that through publishing the *PISA Assessment Framework and Pedagogical Practice Series*, together with the associated series of teacher professional development programmes, teachers will have a better understanding of the 'what' (i.e. the elements and structure of instructional content) and 'how' (i.e. theory informed, technique of design) of assessment as exemplified in PISA. Teachers will also learn how to evaluate students' learning effectively base on the assessment results.

The current series is prepared particularly for teachers of Chinese/English languages, Mathematics, Science and Liberal Studies. It is hoped that, through engaging in our professional development activities, teachers will develop their repertoire of assessment. We encourage professional sharing among teachers. Therefore, teachers participated in our programmes are welcomed to share their works and insights by posting comments and constructed items onto HKPISA Centre's website.

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The Assessment of Mathematical Literacy Learning from PISA

1 Definition of Mathematical Literacy

The PISA assessment of mathematical literacy is concerned with the capacities of students to analyse, reason, and communicate ideas effectively as they pose, formulate, solve, and interpret mathematical problems in a variety of situations. Therefore, the assessment focuses on real-world problems, moving beyond the kinds of situations and problems typically encountered in school classrooms.

PISA mathematical literacy deals with the extent to which 15-year-old students can be regarded as informed, reflective citizens and intelligent consumers. PISA defines mathematical literacy as "an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgments and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen."

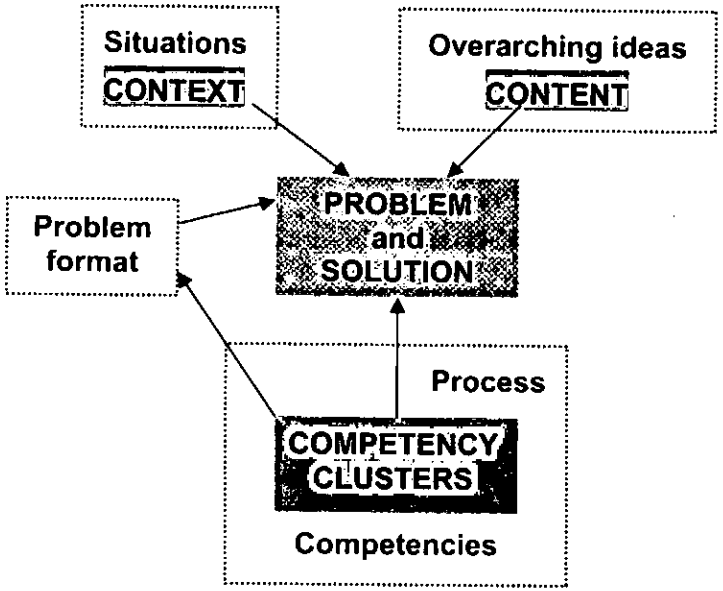
2 Organisation of the Domain

The PISA mathematics framework provides the rationale for, and the description of, an assessment of the extent to which 15-year-old students can handle mathematics in a well-founded manner when confronted with real-world problems. Three components are distinguished to describe the mathematical literacy domain:

- The *situations* or *contexts* in which the problems are located.
- The *mathematical content* that has to be used to solve the problems, organised by certain *overarching ideas*.
- The *competencies* that have to be activated in order to connect the real world, in which the problems are generated, with mathematics in order to solve the problems.

Figure 2.1 depicts the relationships between these components. The extent of a person's mathematical literacy is seen in the way he or she uses mathematical knowledge and skills in solving problems. Problems may occur in a variety of situations or contexts within the experience of an individual. Within a situation, a problem has a more specific context. Another component is the mathematical content that a person might bring to bear in solving a problem. From the overarching ideas, the content used in solving a problem is extracted. The arrows going from the context and content to the problem show how the real world makes up a problem. The mathematical processes that students apply as they attempt to solve problems are referred to as mathematical competencies. The particular competencies needed to solve a problem are related to the nature of the problem. This interaction is represented by the arrow from the competency clusters to the problem and its solution. The competencies employed in solving a problem could also be related to the form of the problem and its precise demands. This path is represented by the remaining arrow going from the competency clusters to the problem format. These three components involved in mathematical literacy will be described in more detail in the following sections.

Figure 2.1 Components of the mathematics domain



2.1 Situations and Contexts

An important aspect of mathematical literacy is engagement with mathematics: using and doing mathematics in a variety of situations. It has been recognised that in dealing with issues that lend themselves to a mathematical treatment, the choice of mathematical methods and representations is often dependent on the situations in which the problems are presented. The context of an item is its specific setting within a situation that resembles real-life setting. Four situation types are defined and used in PISA to assess students' literacy:

- Personal
- Educational/occupational
- Public
- Scientific

The context of an item is its specific setting within a situation. It includes all the detailed elements used to formulate the problem.

2.2 Mathematical Content

Based on the general conception of mathematics and the major threads comprising school mathematics curriculum, four overarching ideas are identified in PISA:

- **Space and shape** relates to spatial and geometric phenomena and relationships, drawing on the curricular strand of geometry.
- **Change and relationships** involves mathematical manifestations of change as well as function relationships and dependency among variables, which relates most closely to the curricular strand of algebra.
- **Quantity** involves the understanding and processing of numeric phenomena as well as quantitative reasoning.
- **Uncertainty** involves the concepts of data, statistics and probability.

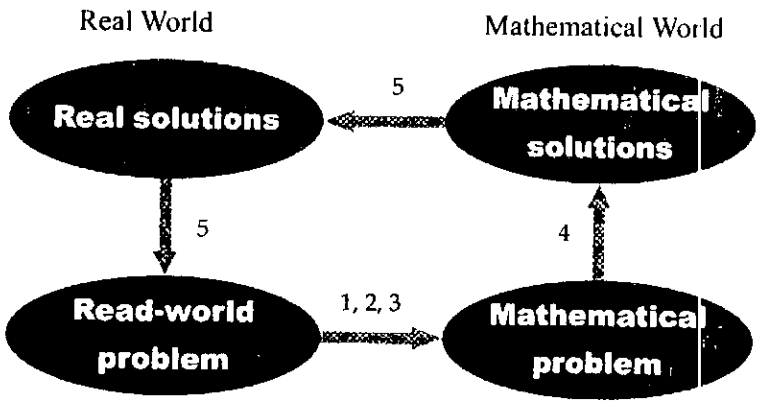
Each overarching idea represents a certain perspective, which can be thought as possessing a core and somewhat blurred outskirts that allow for intersection with other overarching ideas. In principle, any overarching idea may intersect any other overarching idea.

2.3 Mathematical Processes

PISA refers to the fundamental process that students use to solve real-life problems as "mathematisation". This process could be described in five steps (and depicted in Figure 2.3.1):

- Starting with a problem situated in reality
- Organising it according to mathematical concepts and identifying the relevant mathematics
- Transforming the real-world problem into a mathematical problem
- Solving the mathematical problem
- Making sense of the mathematical solution in terms of the real-life situation, including identifying the limitations to the solution

Figure 2.3.1 The mathematisation cycle



Individuals who could engage successfully in mathematisation need to possess a number of mathematical competencies. Different parts of mathematisation draw differently upon these competencies. Eight characteristic mathematical competencies are examined in PISA:

- Thinking and reasoning
- Argumentation
- Communication
- Modelling
- Problem posing and solving
- Representation
- Using symbolic, formal and technical language and operations
- Use of aids and tools

These eight competencies overlap among each other, and it is usually necessary to draw simultaneously on many of the competencies. In order to describe the competencies in a comprehensible and manageable way, these cognitive abilities are grouped into three clusters of competencies, based on the kinds of cognitive demands needed to solve different mathematical problems.

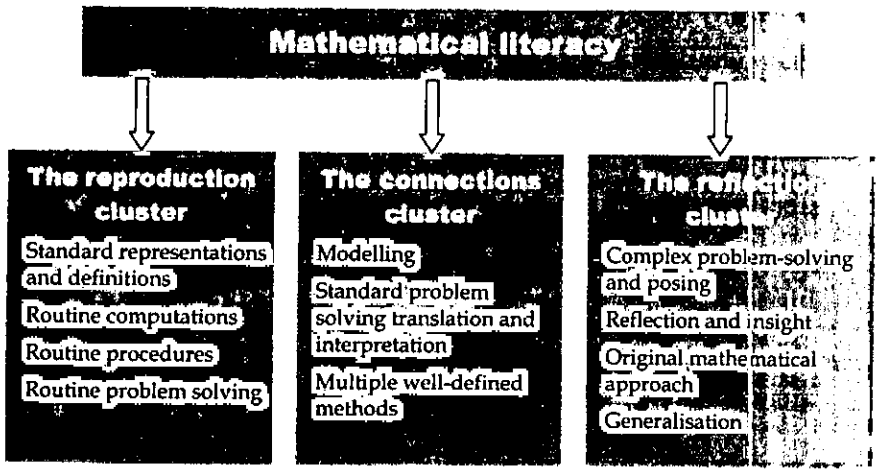
- **Reproduction cluster** involves reproduction of practised knowledge, recollection of familiar mathematical objects and properties, performance of routine operations, and application of standard

algorithms.

- **Connections cluster** involves integration and connection of practised materials. The *connections* cluster competencies build on the *reproduction* cluster competencies in taking problem solving to situations that are not routine.
- **Reflection cluster** includes advanced reasoning, argumentations, abstraction, generalisation and modelling in problem settings that contains more unfamiliar elements than those in the *connections* cluster.

Figure 2.3.2 summarises the distinctions between the three competency clusters.

Figure 2.3.2 Diagrammatic representation of the competency clusters



2.4 Assessing Mathematical Literacy - Task Characteristics

Items are designed to relate predominantly to the overarching ideas, embody one or more of the mathematical processes, and identify predominantly with one of the competency clusters. PISA assesses mathematical literacy through a combination of items with different format types:

- Multiple-choice items
- Complex multiple-choice items
- Close-constructed response items
- Short response items
- Open-constructed response items.

Based on past experience, the multiple-choice type is generally regarded as most suitable for assessing items that would be associated with the *reproduction* and *connections* competency clusters. Closed-constructed response items pose questions similar to multiple-choice items but require higher-order goals and more complex processes. Open-constructed response items involve higher-order cognitive activities and often require students to show the steps taken or to explain how the answer is reached.